GEARS AND GEAR-CUTTING

EDITED BY

F. J. CAMM

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GEARS AND GEAR-CUTTING

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GEARS AND GEAR-CUTTING

PRINCIPLES, PRACTICE AND FORMULÆ

EDITED BY

F. J. CAMM

Editor of "*Practical Engineering*"

WITH OVER 100 ILLUSTRATIONS

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PREFACE

GEAR design and methods of cutting gears have progressed considerably during the last thirty years. Accurate gears are so essential to a wide variety of machine tools and mechanisms that the time seemed opportune to produce this practical work dealing with gear design, types, functions and methods of cutting them, together with formulæ necessary for the designer and the operator.

Additional material deals with the making of end-mills, hobs, and gear-generating cutters. The old method of cutting spur gears by means of a rotary milling cutter, one tooth at a time, is still used in some shops and, therefore, is included in this book. Gear measurement is an important aspect of inspection, which is also dealt with.

This book includes the articles on gear-cutting contributed to *Practical Engineering* by W. A. Tuplin, D.Sc.

F. J. CAMM.

CONTENTS

CHAPTER		PAGE
I.	TYPES, CUTTING METHODS AND TERMS	9
II.	BEVEL GEARS	29
III.	WORM GEARS	34
IV.	GEAR GENERATION	41
V.	GEAR-WHEEL FORMS	66
VI.	EPICYCLIC GEAR TRAINS	71
VII.	METHODS OF MOUNTING	74
VIII.	MEASURING GEARS	78
IX.	HOBS, END-MILLS AND GENERATING CUTTERS	100
X.	LOAD CAPACITY OF GEARS	113
XI.	THE EFFICIENCY OF GEARS	117
XII.	USEFUL FORMULÆ FOR GEARS	119
	INDEX	141

GEARS AND GEAR-CUTTING

CHAPTER I

TYPES, CUTTING METHODS AND TERMS

THE toothed gear has long been a familiar element in mechanical engineering and its importance seems likely to increase as development proceeds. Early examples of gears were cumbrous, noisy and inefficient, owing to the lack of suitable materials, design technique and manufacturing facilities. Improvements in these respects have more than kept pace with advancing demands and toothed gearing is now confidently employed at speeds and power outputs that could not have been attempted a few decades ago.

Gears are also used to transmit power between rotating shafts which are not in the same straight line and which have, in general, different speeds. Other means are possible in some circumstances, but where a compact drive is

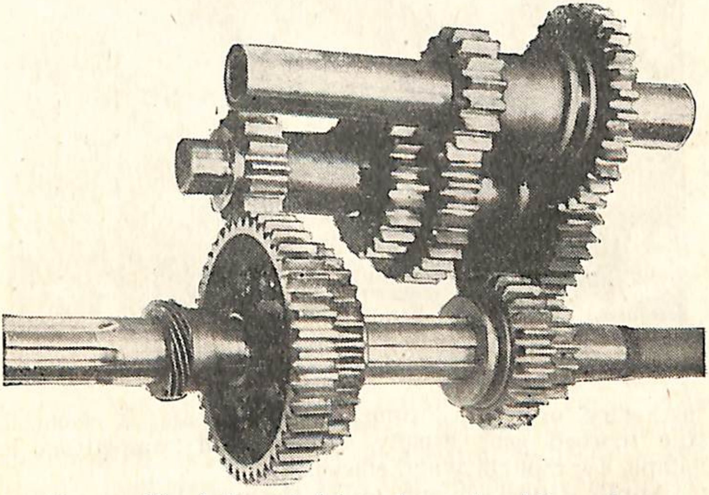

FIG. 1.—The shafts and pinions of an automobile gearbox.

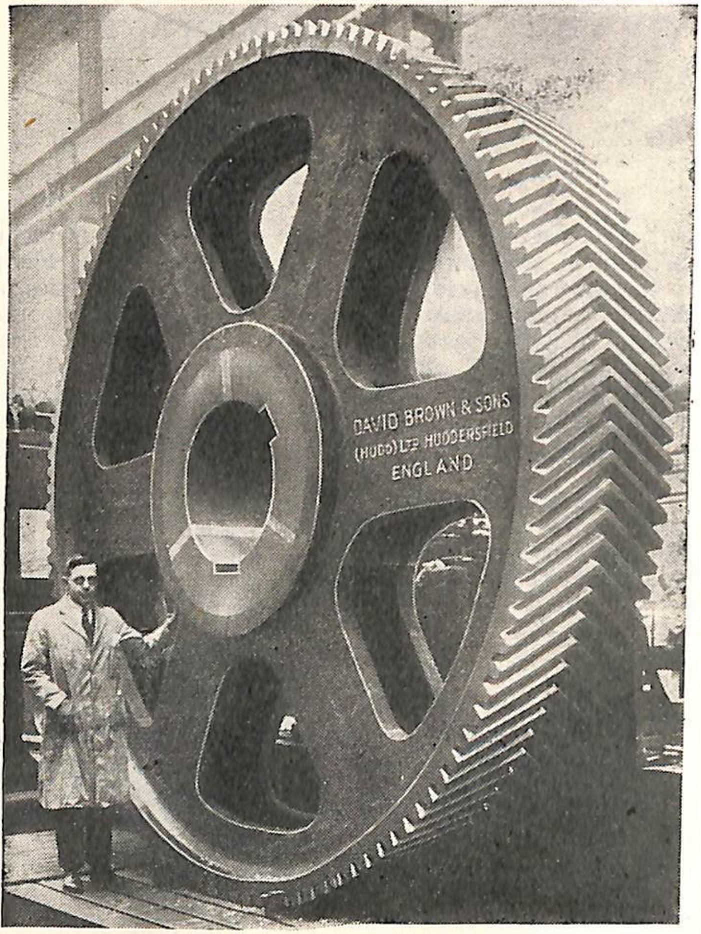
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FIG. 2.—This type is known as the double-helical gear with continuous teeth.

necessary, or where "timing" of the shafts is essential, the toothed gear usually surpasses all competitors in simplicity, reliability and efficiency.

Applications.—Examples of the use of gearing occur in the conventional automobile. For example, the camshaft which operates the valves has to run at half the crankshaft speed, and in many instances it is driven from the crankshaft through gearing.

In order to permit the engine to run within a reasonable economical speed-range whilst driving the vehicle at different speeds, the gearbox contains several pairs of gears, the drive being taken through any particular pair at will, to suit the conditions under which the vehicle is running.

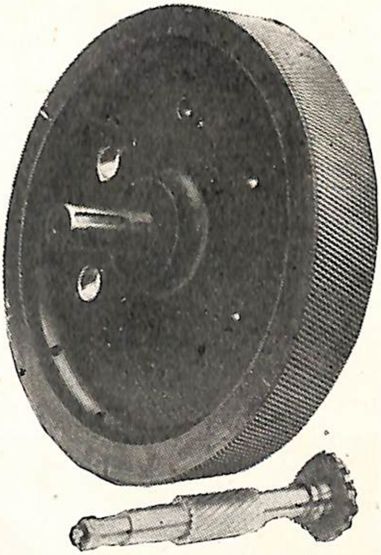


FIG. 3.—An example of a single-helical gear.

From the propeller shaft which runs parallel to the length of the vehicle, the drive is taken to the rear axle through another pair of gears, in which there is a reduction in speed besides a change in the direction of the drive. The rear axle gear assembly also contains the "differential," which is a gearing assembly designed to divide the driving effort equally between the two rear wheels.

In contrast with the small gears used in automobiles, gearing is used in diameters up to 20 ft., for example, in ship propulsion, where the economical speed of the steam turbine may be about 3000 r.p.m., and that of the propeller about 250 r.p.m.

In industrial service, gearing is now very extensively used, largely because of the widespread application of electric-motor drives. The economical running speeds of electric motors of moderate output range from about 750 r.p.m. to 1500 r.p.m., whereas the greater proportion of industrial machinery runs at considerably lower speeds, usually from about 50 to 200 r.p.m. In such applications, toothed gearing forms a compact and efficient transmission, whose availability makes it possible to design driving and driven machines each to run at its own most economical speed.

Main Types of Gear.—Toothed gearing is first of all divided on the basis of relative shaft position. The simplest examples are those used for parallel shafts as, for instance, in the conventional clock or in the automobile gearbox (see Fig. 1). These are classed as spur gears if their teeth are parallel to the shaft centre lines, otherwise they are helical gears which may be either "single-helical" or "double-helical" (Figs. 3 and 2). Triple-helical teeth (Fig. 4) are also occasionally used but are not to be recommended.

If the shaft centre lines intersect, the bevel gear is employed. Here again there is a subdivision associated with tooth formation, "straight bevel" gears having teeth whose centre lines converge at the apex, *i.e.* the

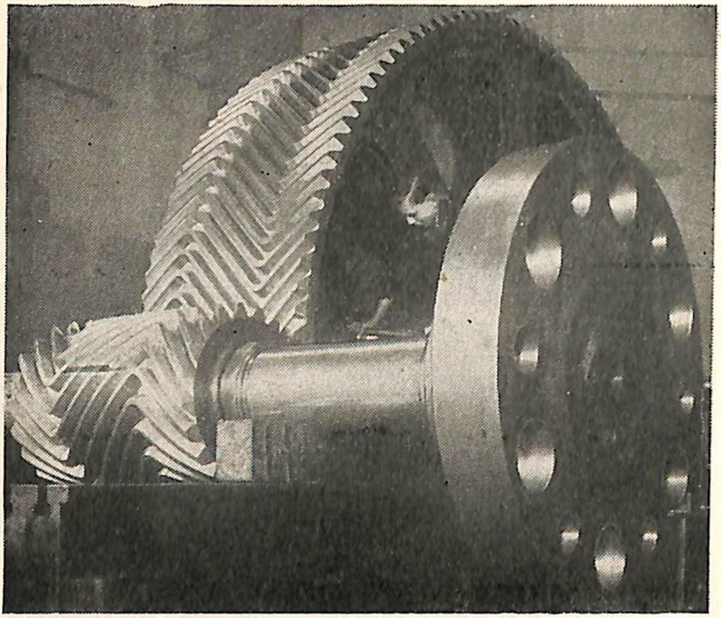


FIG. 4 —Triple-helical teeth are occasionally used, but are not to be recommended.

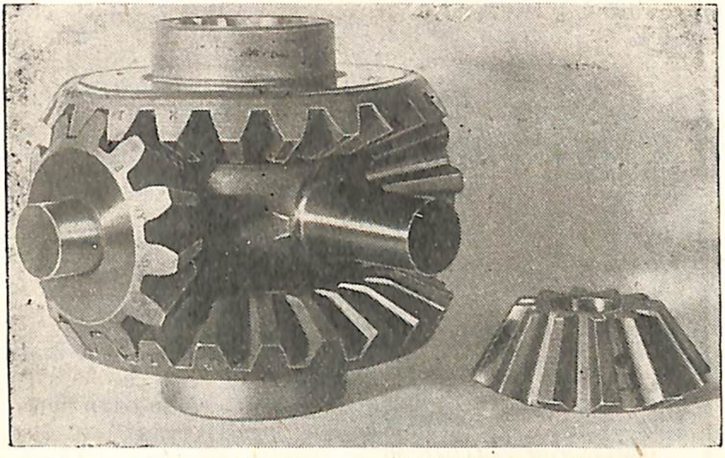


FIG. 5.—An example of gears in which the centre lines of the teeth converge towards the apex (straight bevel).

intersection of the shaft centre lines (see Fig. 5). When, as in Fig. 7, the tooth centre lines are set at an angle to the straight lines which converge at the apex, the gear is classed as a " spiral bevel " or as a " single-helical bevel." Bevel gears are usually applied to shafts whose centre lines are perpendicular to one another, but they can be manufactured to suit almost any shaft angle.

If the shaft centre lines are not parallel and do not intersect, the connection between the shafts may be effected by spiral gears (Fig. 6), which have, however, a comparatively small load capacity. Each gear in such a combination is actually a helical gear, but the fact that the shaft centre lines are not parallel is the distinction that leads to the description " spiral gears." It also causes the contact between any pair of mating teeth to be confined to a theoretical point, this being the reason for the small load capacity.

Of a pair of gears, the one with the smaller number of teeth is called the " pinion " and the other is the " wheel."

By forming one member of a pair of spiral gears so that the teeth " envelop " the other gear, line contact between pairs of mating teeth is secured and the worm gear (Fig. 8) is produced. The gear of parallel formation is called the "worm." Its teeth are helical and are called "threads," and it may have (and often has) only one thread. If the worm has more than one thread, the number of threads is often described as the "number of starts." This is to avoid confusion with screw-thread terminology. Occasionally the worm is made "hollow faced" so as to envelop the wheel which also envelops the worm, but this "hour-glass" worm gear has no advantage that justifies the extra cost and complication involved in manufacturing it.

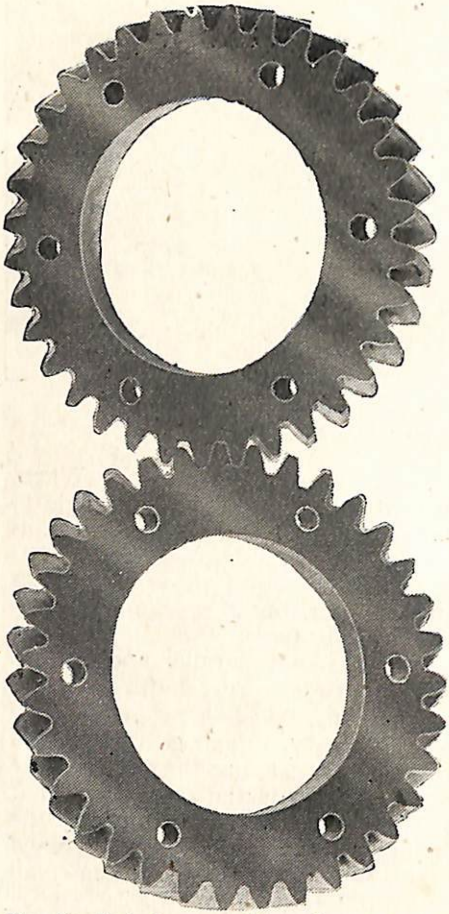


FIG. 6.—Spiral gears. These have a comparatively small load capacity.

Essential Requirements.—The first essential of any pair of gears is that of uniform angular velocity transmission. This means that for each degree of angular movement of the driving gear the mating gear rotates through a certain multiple of one degree.

The second is that despite the distortion of the teeth under load, the gears must be capable of

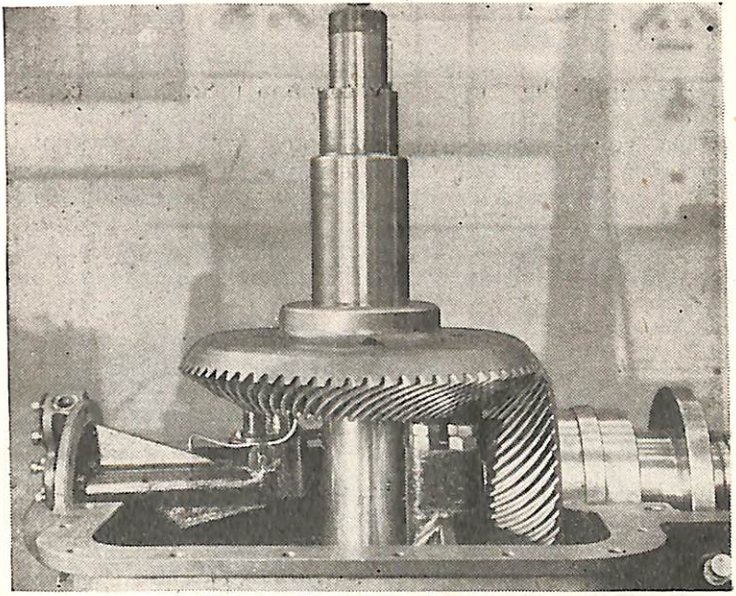


FIG. 7.—A spiral bevel-gear unit.

transmitting power without appreciable shock or noise.

The third requirement is that the gears must transmit the full rated torque without fear of tooth breakage and without showing signs of excessive wear on the working surfaces of the teeth.

Temperature rise resulting from heat generation by friction must be kept within due limits and, particularly with high-speed gears, this may demand special cooling arrangements.

Uniform Angular Velocity Transmission.—Smooth cylinders mounted on shafts and pressed into contact with each other will rotate at relative speeds depending on their diameters, and will transmit power up to the relatively low limit which causes slip to occur. Toothed gears are required to operate in the same way as such "rolling cylinders" except that the intermeshing of the teeth prevents any possibility of slip. The rolling cylinders corresponding to any pair of toothed gears are called the "pitch cylinders." Fig. 9 shows the pitch cylinders corresponding to a pair of helical gears, the flat strip representing the "imaginary common rack." The corresponding geometrical forms in the case of bevel gears are cones (see Fig. 10) with a disc representing the imaginary common crown-wheel. A worm-wheel has a corresponding pitch cylinder which rolls, however, with a plane and this is the "pitch plane" associated with the worm. A worm has no pitch cylinder, properly so called, although it does contain an imaginary cylinder which is a useful basis on which to work in calculating detail dimensions of the worm; it may be regarded as a "nominal pitch cylinder," and is indicated in Fig. 11.

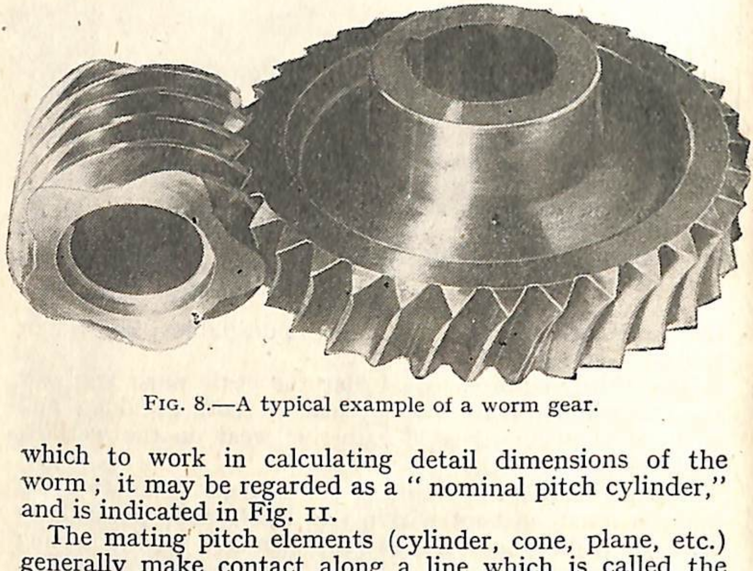


FIG. 8.—A typical example of a worm gear.

The mating pitch elements (cylinder, cone, plane, etc.) generally make contact along a line which is called the "pitch line"; in the case of spiral gears the contact is confined to a single "pitch point."

It can be shown that if a pair of gears is to transmit uniform angular velocity the line which lies perpendicular to the tooth surfaces at any point of contact must intersect the pitch line. Many geometrical forms will meet this requirement, and choice amongst them is influenced by ease of production with the necessary degree of accuracy.

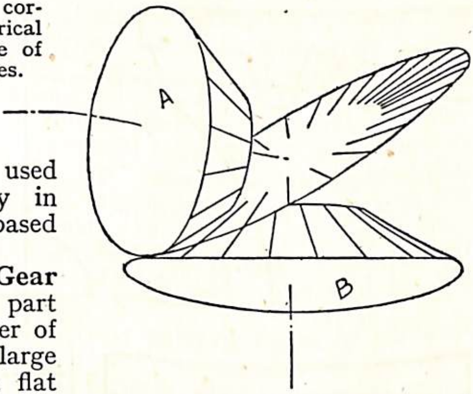


FIG. 10 (*right*).—The corresponding geometrical forms in the case of bevel gears are cones.

The tooth form used almost exclusively in modern practice is based on the involute.

Generation of Gear Teeth.—A small part of the pitch cylinder of a gear of very large size appears as a flat surface. If the gear were of infinite size, a small part of it would have a flat "pitch surface"; such a gear is known as a rack.

A rack may mesh with a gear (usually called a pinion in this connection), and rota-

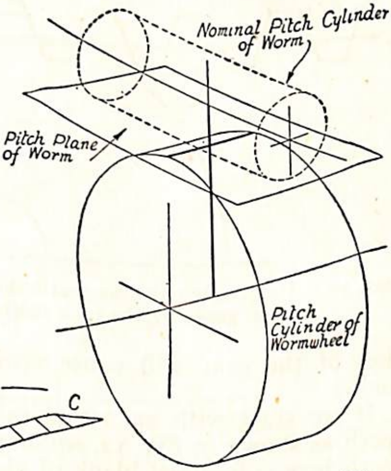

Nominal Pitch Cylinder of Worm
Pitch Plane of Worm
Pitch Cylinder of Wormwheel

FIG. 11 (*above*).—The "nominal pitch cylinder" of a worm.

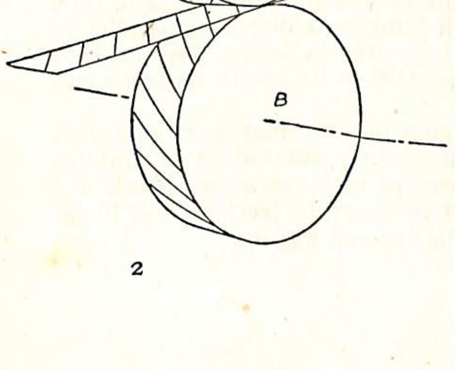


FIG. 9 (*left*).—A diagrammatic sketch, showing the pitch cylinders corresponding to a pair of helical gears.

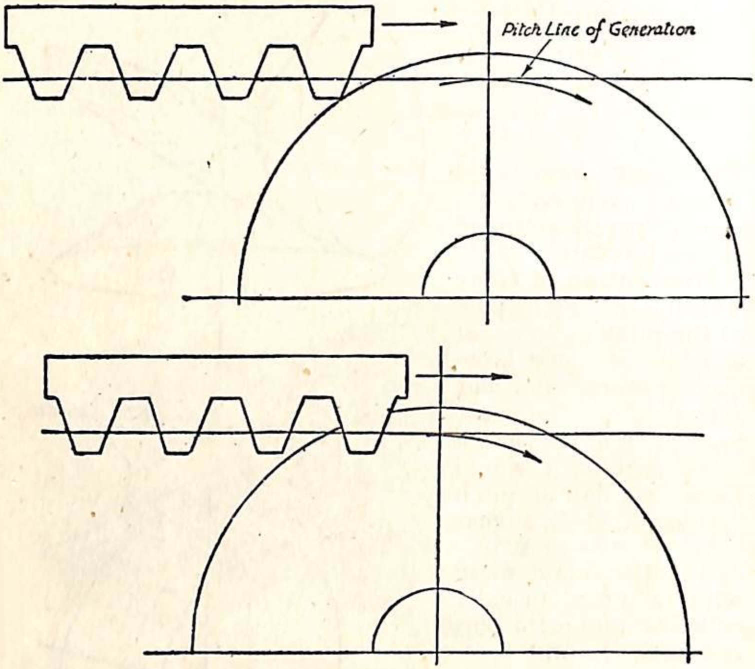


Fig. 12.—Progressive diagrammatic sketches illustrating the generation of gears by the rack method (*see also* Fig. 13).

tion of the gear will cause straight-line motion of the rack.

If we start with an actual rack, having straight-sided teeth as shown in Fig. 12, we may use this to " generate " teeth in a cylindrical blank of plastic material. The rack may be set with its teeth lying in a direction parallel to the axis of the blank and situated in such a position that when the rack is moved parallel to its length its teeth will penetrate the blank.

Now if the rack be given a uniform motion of this sort, and the blank be simultaneously rotated at a uniform rate, the relative movement of rack teeth and blank will cause the teeth to mould or generate teeth in the blank in the manner shown in Fig. 12 and Fig. 13.

It can be proved that, within certain limitations, the teeth generated in this manner are of involute form.

The Involute.—Fig. 14 represents the end view of a cylinder (radius r_o) from which a string AB is being unwrapped, the end B having been originally at C. The curve CB traced by the end of the string is an involute to the circle, which is called the "base circle" of the involute.

When an involute gear is generated from a rack in the manner already described, the size of the base circle associated with the involute tooth profiles is determined by the relative motions of rack and blank and by the spacing of the rack teeth.

The distance between corresponding flanks of adjacent rack teeth measured in a direction parallel to the plane containing the tips of the teeth is called the normal pitch.

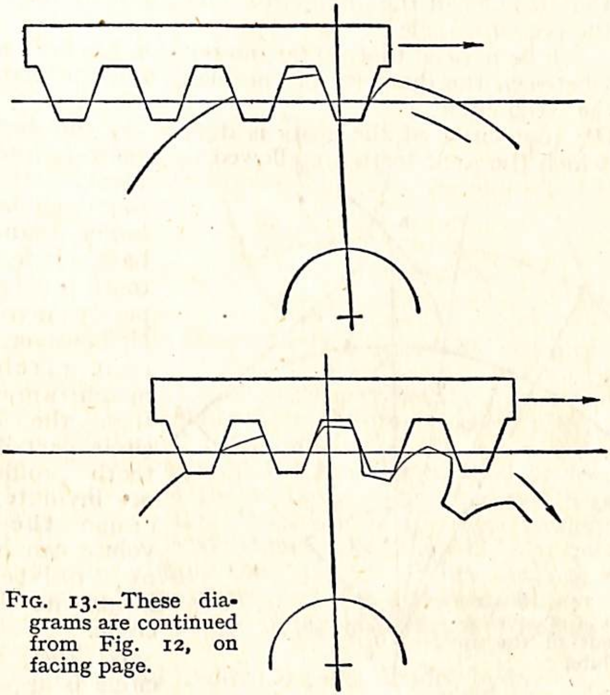


Fig. 13.—These diagrams are continued from Fig. 12, on facing page.

The distance between corresponding flanks of adjacent rack teeth measured in the direction perpendicular to the flanks is called the " base pitch."

If the gear blank, during generation, makes one revolution whilst the rack is moved through a distance equal to t normal pitches, the gear produced will have t teeth. The base circle of the involute profiles will be the one whose circumference is equal to t times the base pitch of the rack teeth.

The " pitch circle of generation " of the gear will be the circle whose circumference is equal to t times the linear pitch of the rack.

The angle between a line drawn perpendicular to the flank of the rack tooth and a line joining the tips of the rack teeth is called the " pressure angle," and it can be shown that diameter of the base circle of the gear is equal to the diameter of the pitch circle multiplied by the cosine of the pressure angle.

It will be noticed that, so far, no relation has been specified between the diameter of the blank and the diameter of the pitch circle.

The root circle of the gears is defined by the distance by which the rack teeth are allowed to penetrate into the blank. If the root circle is larger than the base circle, the tooth profiles are purely involute. If, however, the root circle is much smaller than the base circle, part of the tooth profile is not involute, because the involute can have no existence inside its base circle.

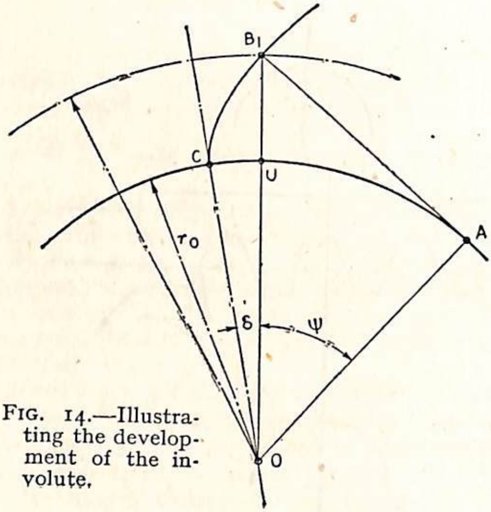


FIG. 14.—Illustrating the development of the involute.

If the root circle is appreciably smaller than the base circle, the part of the tooth which lies inside the base circle is liable to be "undercut"; it is mechanically weak and liable to produce unsatisfactory meshing of the gear with its mating gear.

Tooth Proportions.—On the British Standard System, the total depth of a tooth from tip to root is made 0·716 times the normal pitch of the tooth. The depth to which the tip of a gear tooth penetrates into the space between adjacent teeth in the mating gear is 0·636 times the pitch. The difference between these fractions, *i.e.* 0·080 times the pitch, is called the "clearance."

The distance from the circumference of the pitch circle

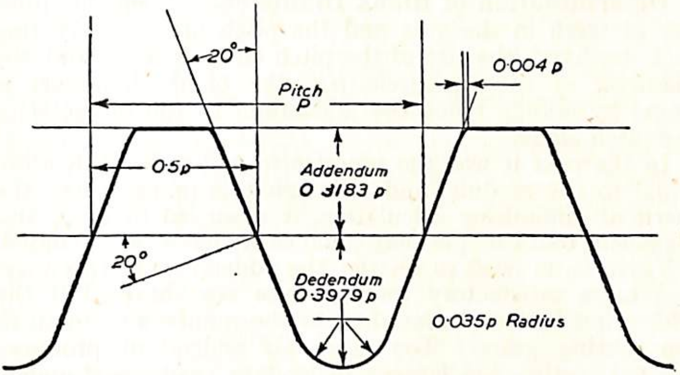


FIG. 15.—The British Standard basic rack-form, 20° pressure angle.

of generation to the pitch circle is called the "addendum" of tooth. The distance from the pitch circle to the root circle is called the "dedendum."

Methods of Defining Pitch.—The method of describing the spacing of the teeth by specifying the distance between them on the pitch circle is the most straightforward one. This is sometimes called the "circular pitch," or more often simply the "pitch." When dealing with helical gears, "transverse pitch" is a more suitable term.

Other methods of specifying pitch are by module and diametral pitch.

Module is equal to circular pitch divided by π.

The advantage of defining pitch in this way is that the pitch diameter of a gear is simply the product of module and number of teeth. This system of measurement has been almost confined to continental practice, using the millimetre as the unit of length, but it is equally applicable on the basis of the inch.

Diametral Pitch.—This is equal to π divided by the circular pitch. The pitch diameter of a gear is equal to the number of teeth divided by the diametral pitch. For most practical purposes it is possible to use a diametral pitch which is a whole number, whereas with the inch as the unit of length, pitch and module are very often fractional.

Table I. gives standard pitches expressed in all three ways.

Determination of Blank Diameter.—When the number of teeth in the gear and the pitch of the generating rack are fixed, the size of the pitch circle is fixed and the diameter of the tip circle (*i.e.* the blank diameter) is found by adding twice the addendum to the diameter of the pitch circle.

In the past it was the practice to make the addendum equal to the module, and although this practice had the merit of minimising calculation, it often led to weak and inefficient tooth shapes that could easily have been avoided.

There is no need to restrict the addendum in this way, and more satisfactory tooth shapes are obtained if the addendum is varied according to the numbers of teeth in the mating gears. Too large an addendum produces pointed teeth; too large a dedendum produces "undercut" teeth with weak roots. For a pressure angle of 20 degrees and a tooth depth equal to 0·716 times the pitch, addenda defined by the following formulæ lead to satisfactory tooth shapes.

$$\text{Addendum of pinion} = \left(1{\cdot}4 - 0{\cdot}4\frac{t}{T}\right)\frac{p}{\pi}$$

$$\text{Addendum of wheel} = \left(0{\cdot}6 + 0{\cdot}4\frac{t}{T}\right)\frac{p}{\pi}$$

where t = number of teeth in pinion.
T = number of teeth in wheel.
p = pitch of cutter.

In each case

$$\text{Dedendum} = 0{\cdot}716p - \text{Addendum}.$$

Fig. 15 represents the British Standard basic rack-form, and Figs. 16 and 17 show typical involute teeth generated from it.

The greater the number of teeth in a gear, the greater is the size of the base circle for a given pitch, and as the depth of tooth is the same in all cases, it becomes only a small fraction of the involute when the number of teeth is very large. Consequently, gears of large numbers of teeth have profiles which are only slightly curved, and the tooth of the rack (which is really an infinitely large gear) has no curvature at all, *i.e.*, it is straight-sided as already mentioned.

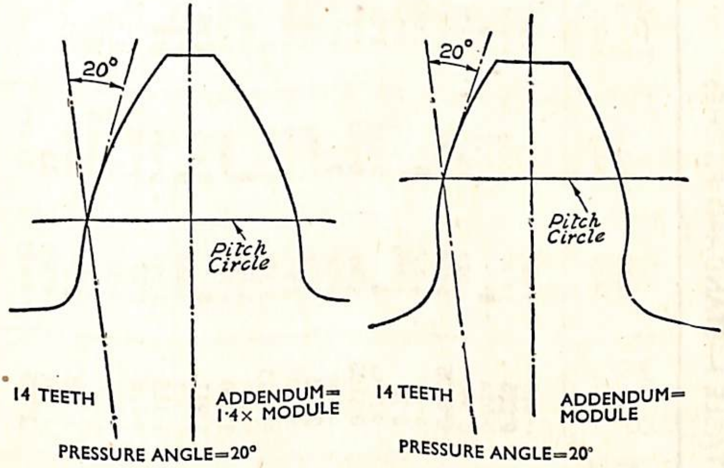


FIG. 16.—Typical involute teeth generated from the British Standard rack-form.

Generating Rack.—The fundamental principle involved in moulding a gear in the manner previously described is used in nearly all modern gear-cutting processes. The generating member is made in the form of a cutter, and besides its moulding motion has another which causes its teeth to cut.

In some cases the cutter itself is actually of rack-form. In other cases the cutter corresponds to some particular form of gear, and in the process of generating the teeth,

TABLE I—STANDARD PITCHES

Circular Pitch (in.).	D.P. (in.).	Module (mm.).	Circular Pitch (in.).	D.P. (in.).	Module (mm.).	Circular Pitch (in.).	D.P. (in.).	Module (mm.).
0·1855	16·933	**1·5**	0·618	5·080	**5**	1·396	**2·25**	11·286
0·1875	16·755	1·516	**0·625**	5·026	5·053	1·484	2·117	**12**
0·1963	**16**	1·587	0·628	**5**	5·077	**1·5**	2·094	12·127
0·2164	14·514	**1·75**	0·680	4·618	**5·5**	1·571	**2**	12·701
0·2244	**14**	1·814	**0·6875**	4·569	5·558	1·608	1·954	**13**
0·247	12·700	**2**	0·742	4·233	**6**	**1·625**	1·933	13·138
0·250	12·566	2·021	**0·75**	4·189	6·064	1·732	1·814	**14**
0·262	**12**	2·118	0·7854	**4**	6·350	**1·75**	1·795	14·148
0·278	11·288	**2·25**	**0·8125**	3·867	6·569	1·795	**1·75**	14·512
0·309	10·160	**2·5**	0·866	3·628	**7**	1·855	1·693	**15**
0·3125	10·053	2·527	**0·875**	3·590	7·074	**1·875**	1·676	15·159
0·314	**10**	2·540	0·898	**3·5**	7·260	1·978	1·587	**16**
0·340	9·236	**2·75**	0·9375	3·351	7·580	**2**	1·571	16·170
0·371	8·466	**3**	0·990	3·175	**8**	2·094	**1·5**	16·930
0·375	8·377	3·032	**1·000**	3·142	8·085	2·226	1·412	**18**
0·393	**8**	3·177	1·047	**3**	8·465	**2·25**	1·396	18·191
0·433	7·257	**3·5**	1·113	2·822	**9**	2·474	1·270	**20**
0·4375	7·181	3·537	**1·125**	2·792	9·095	**2·5**	1·257	20·212
0·449	**7**	3·630	1·142	**2·75**	9·233	2·513	**1·25**	20·317
0·495	6·350	**4**	1·237	2·54	**10**	**2.75**	1·142	22·233
0·5	6·283	4·042	**1·25**	2·513	10·106	**3**	1·047	24·254
0·524	**6**	4·236	1·257	**2·5**	10·163	3·142	**1**	25·400
0·557	5·644	**4·5**	1·361	2·309	**11**			
0·5625	5·585	4·547	**1·375**	2·285	11·117			

the cutter and the gear are rotated just as the corresponding gears would actually do when in mesh.

In all cases, however, it is convenient to examine the conditions of generation and meshing of gears as if a rack were present. Actual tooth forms are most easily determined from that of the imaginary basic rack.

Meshing of Double-Helical Gears.—If two helical gears are generated by the same basic rack, and have equal and opposite spiral angles, they may be meshed together with their shafts parallel. On any plane perpendicular to the axes of the gears, the tooth action is exactly the same as in a pair of spur gears. At any instant the teeth in one transverse plane are in a different phase of engagement from those in other transverse planes. If the width of the gears is great enough, every phase of engagement is occurring in some transverse plane or other. Consequently, helical gears operate much more smoothly than do spur gears of comparable dimensions.

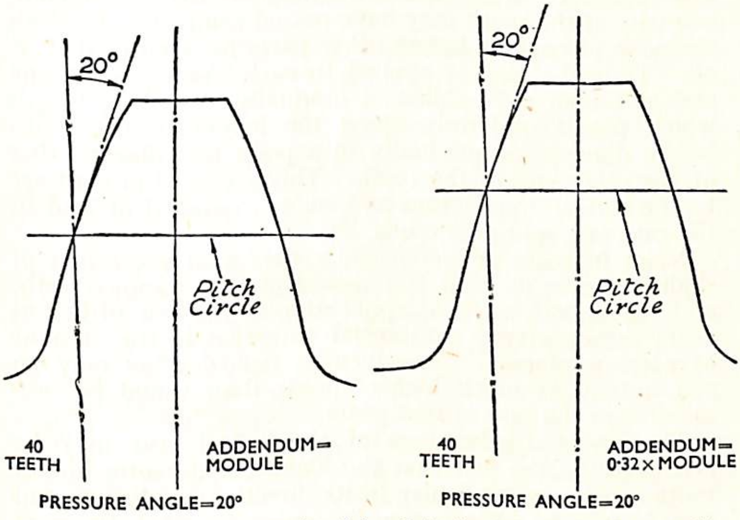


FIG. 17.—Another example of involute teeth generated from the British Standard rack-form.

This can be appreciated also by considering how the load is applied to the tooth of a rotating gear. In the case of spur gears the full width of the tooth enters the zone of engagement at some particular instant, so that load is applied to the tooth over its whole width at once. Similarly, when the tooth passes out of the zone of engagement the load is removed instantaneously.

In a pair of helical gears, however, one end of the tooth enters the zone of engagement first, and as it passes through, other sections of the tooth enter the zone, one after another. If the width of the gears is great enough, one part of the tooth may have passed completely through the zone of contact before other parts have entered it at all. Thus the load is applied to each tooth first at one end and then over a line of gradually increasing length which passes uniformly along the length of the tooth before diminishing gradually to a point and disappearing at the other end of the tooth. This action is in contrast to the sudden application and sudden removal of load in the case of a spur-gear tooth.

Noise in gears is produced by the rapid succession of slight impacts as load is imposed on the various teeth, and the smooth mode of application in the case of helical gears causes a very substantial reduction in the amount of noise produced. Consequently, helical gears may be run quietly at much higher speeds than would be permissible in the case of spur gears.

The essential advantage of the helical gear may be attributed to the fact that the longitudinal centre line of tooth is not perpendicular to its direction of motion, and this feature is advantageously copied in other types of gear.

Overlap of Helical-Gear Teeth.—In order to make sure that all phases of engagement do occur simultaneously, it is necessary that one end of any tooth should overlap, or should at least come in line with the opposite end of the adjacent tooth. The condition for this is that the face-width should be at least equal to the transverse pitch multiplied by the cotangent of the spiral angle. If for any reason the face width cannot be made so great as this, the smoothest possible action will not be obtained, but the gears will, nevertheless, be superior in action to spur gears.

End-Thrust.—The effective force exerted by one gear tooth on the mating tooth is perpendicular to the centre lines of the shafts. If the teeth are of helical form, however, equal and opposite forces are exerted on the two gears in the direction parallel to the shaft centre lines. These forces would cause the gears to slide axially out of mesh if nothing were done to prevent it, and consequently a thrust bearing is necessary on each shaft.

As in some instances it may be difficult or impracticable to provide suitable thrust bearings, the double-helical form of tooth has been developed. A gear of this type has helical teeth of a particular spiral angle in one half of its face-width, whilst the other half has similar teeth of the opposite hand. Thus the end-thrust produced by tooth load in one half of the gear acts in the opposite direction to that produced in the other half, and if either of the mating gears is free to move axially, it will do so and will take up the position which causes the two end-thrusts to be equalised.

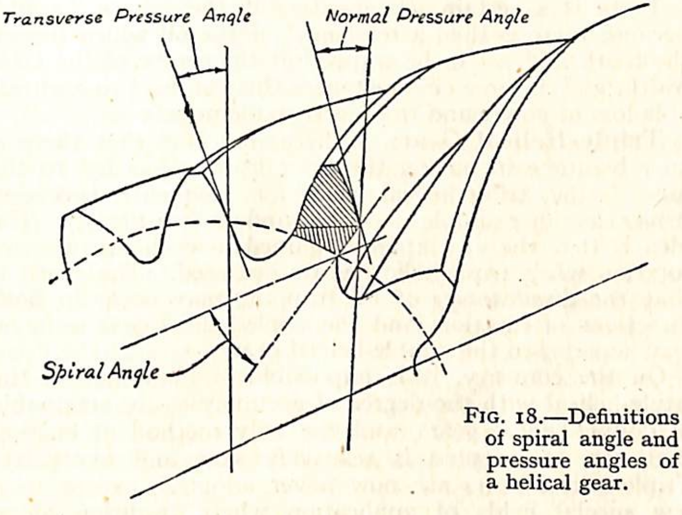


Fig. 18.—Definitions of spiral angle and pressure angles of a helical gear.

Thus, not only is it unnecessary to provide a thrust bearing, but the tooth formation ensures that the total load is equally divided between the two halves of the face-width. Owing to bending and twisting of slender pinions under load, there is not the same certainty in this respect with spur gears or single-helical gears.

Direction of Rotation.—In one direction of rotation, the teeth of a pair of helical gears move with the apex leading, whilst if the direction of rotation is reversed, the apex follows the remainder of the tooth. It is sometimes supposed that there is a particular advantage in one direction of rotation over the other, but this is not the case.

There is a certain *disadvantage* if the apices "trail," because there is then a tendency for the oil which lies on the tooth surfaces to be trapped at the centre of the face-width, and in some circumstances this can lead to appreciable loss of power and to objectionable noise.

Triple-Helical Gears.—The assumption that there is an advantage in having the apex leading has led to the use of the triple-helical gear for frequently-reversing drives, as, for example, to mine winders (see Fig. 4). The idea is that the advantage is gained in each direction of rotation when triple-helical gears are used. The truth is that the *disadvantage* of oil trapping may occur in both directions of rotation, and the triple-helical gear is in no way superior to the double-helical gear.

On the contrary, it is impossible to manufacture the triple-helical with the degree of accuracy easily attainable in double-helical gears, and the only method of cutting that can be adopted is relatively slow and expensive. Triple-helical gears are now never adopted, except in a few special fields of application where tradition alone continues to demand them.

CHAPTER II

BEVEL GEARS

JUST as a pair of spur gears is the toothed equivalent of a pair of rolling cylinders, a pair of bevel gears is the counterpart of a pair of cones which roll together whilst rotating on fixed axes. It may be noted here that since the surface-speed of each cone is zero at its apex, the two cones cannot roll together unless the apices coincide. In other words, the apex of a bevel gear must be at the point of intersection of the shaft centre lines.

The general principles involved in the generation of spur gears apply with slight modifications to the generation of bevel gears. Instead of a basic rack we have an imaginary basic crown-wheel whose teeth are straight-sided, and this corresponds to a flat disc capable of rolling simultaneously with two cones (see Fig. 10).

The pitch of the teeth of the crown-wheel must clearly diminish as the centre is approached and, so that there may be similar tooth action at all points, the depth of tooth is made proportional to the pitch. Thus the pitch and depth of a crown-wheel tooth at any section are each proportional to the distance of that section from the apex. The pressure angle is the same everywhere.

In generating spur gears a rack-shaped cutter reciprocates

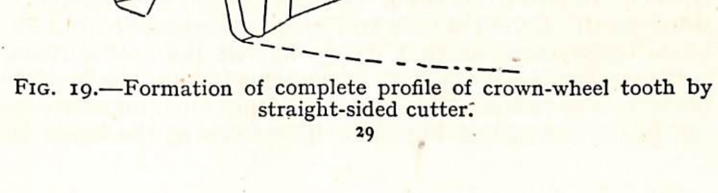


FIG. 19.—Formation of complete profile of crown-wheel tooth by straight-sided cutter.

in the direction of its teeth and produces teeth which are of the same shape and size at all sections. It is not possible, in generating bevel gears, to reciprocate the whole crown-wheel in a similar manner, because different teeth run in different directions. It is not even possible to reciprocate a single complete tooth because depth and thickness have to diminish as the apex is approached. It is possible, however, to sweep out the surface of one flank of a straight-sided tooth of the crown-wheel by a straight-sided cutter blade reciprocated so that its tip follows the root-surface between two teeth of the crown-wheel. As the cutter blade moves towards the apex, its depth of immersion in the blank diminishes, but since the profile of the blade is

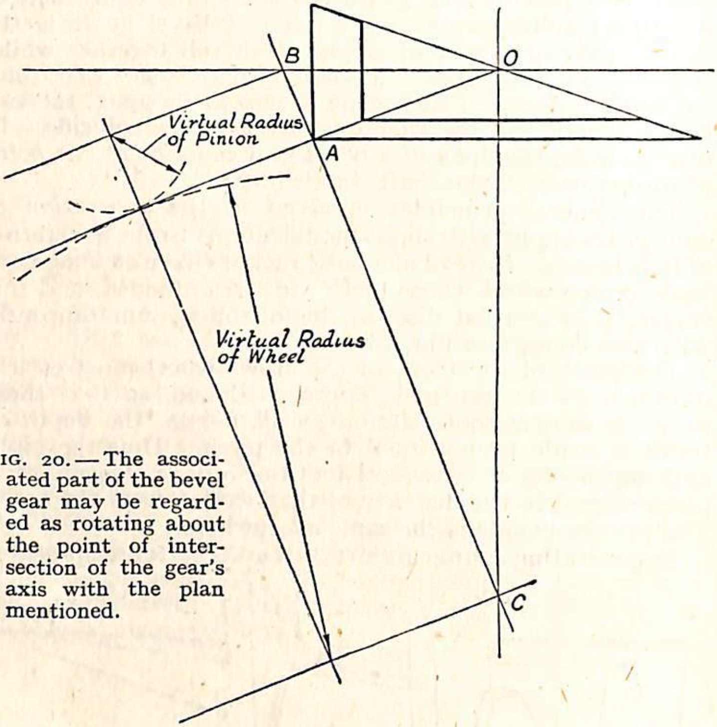


FIG. 20.—The associated part of the bevel gear may be regarded as rotating about the point of intersection of the gear's axis with the plan mentioned.

straight, it produces a complete straight-sided profile everywhere. Here an important virtue of the straight-sided basic crown-wheel tooth is that while the profiles at large and small ends of the tooth are similar to each other, the smaller profile is simply a part of the larger one, so that formation of the entire profile by a single reciprocating cutter is possible. (See Fig. 19.)

The fillet at the root of the tooth must, of course, have the same radius at each end, and in this respect the tooth is not geometrically similar throughout its length, but this is unimportant from the point of view of tooth action.

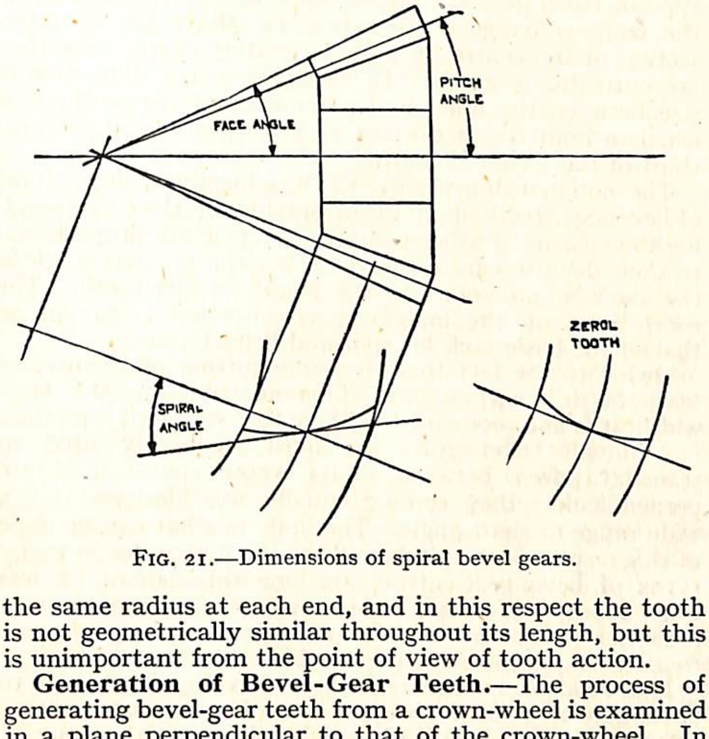


FIG. 21.—Dimensions of spiral bevel gears.

Generation of Bevel-Gear Teeth.—The process of generating bevel-gear teeth from a crown-wheel is examined in a plane perpendicular to that of the crown-wheel. In the neighbourhood of the point of contact of crown-wheel plane and pitch cone of bevel gear, the section of the crown-wheel tooth on the plane mentioned may be regarded as moving in a straight line. The associated part of the bevel gear may be regarded as rotating about the point of intersection of the gear's axis with the plane mentioned (see Fig. 20). The shape of tooth generated in the pinion is the same as that in a spur gear of pitch radius BA, and likewise the wheel-tooth shape is that associated with a spur gear of radius CA. These two radii are the " virtual radii " of the gears. Similar generating action on smaller scales takes place on parallel planes lying nearer to the apex.

Tooth Form of Straight Bevel Gears.—The simplest type of bevel gear is that in which the centre lines of all the teeth converge at the apex, *i.e.* the point of intersection of the centre lines of the mating gears when they are correctly in mesh. To avoid excessive difference in size between the inner and outer ends of the teeth, it is usual to limit the face-width of bevel gears to about one-third of the " cone distance."

The nominal dimensions (pitch, addendum, dedendum) of bevel-gear teeth apply to the outer end ; the corresponding dimensions at other points in the face are proportional to their distances from the apex, but the pressure angle is the same at all points in the length of the tooth. The tooth form of the imaginary crown-wheel is similar to that of the basic rack for spur and helical gears.

Owing to the fact that the same cutting tool generates teeth of different pitch at different points in the face-width, it is not necessary to adhere to " standard " pitches.

Although bevel gears are most frequently used to transmit power between shafts whose centre lines are perpendicular, they can be equally well designed for a wide range of shaft angle. The limit to what can be done in this respect is the pitch angle of the pinion, for in many types of bevel-gear-cutting machine this cannot be less than about 5 degrees. Subject to certain limitations in reduction ratio, shaft angles varying from about 30 degrees to about 150 degrees are permissible.

Like straight spur gears, straight bevel gears are apt to be unduly noisy at high speeds and for the same reason, *i.e.* that the full length of each tooth enters the zone of engagement at the same instant. To overcome the noise difficulty recourse is had to single helical bevel gears or spiral bevel gears.

Spiral Bevel Gears.—If the teeth of a bevel gear have their centre lines not perpendicular to their direction of motion they are called "spiral bevels" or "single-helical bevels." In the latter case the centre line of each tooth is straight but does not pass through the apex. In the spiral bevel gear (Fig. 7) each tooth centre line is a curve, which is usually a circular arc.

In neither type of gear is the spiral angle exactly the same at all points in the face-width. This is not important, provided, of course, that the mating gears have the same spiral angles at corresponding points. The reason for the adoption of the circular arc form for the spiral bevel tooth is that it permits the use of a cutter having a number of blades set in a circular body and that this can operate without reversal of the motion of the cutter. The teeth of straight bevel gears and single-helical bevel gears are generated by reciprocating cutters.

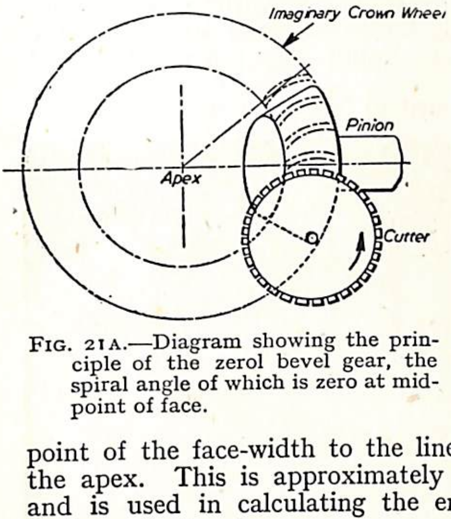


FIG. 21A.—Diagram showing the principle of the zerol bevel gear, the spiral angle of which is zero at mid-point of face.

The nominal spiral angle of spiral bevel gears is the inclination of the centre line of the tooth at the mid-point of the face-width to the line joining that point to the apex. This is approximately the mean spiral angle and is used in calculating the end-thrusts and journal loads on the bearings.

The "Zerol" Gear.—Comparatively recently there has been introduced the "Zerol" spiral bevel gear, so called because although the teeth are of the curved spiral type, the spiral angle at the middle of the face is zero. This confines end-thrust to the amount produced in the corresponding straight bevel gears, but nevertheless gives the smoother running of the spiral bevel gear. The zerol gear thus corresponds to the double-helical gear.

Double-helical bevel gears have been used but the only practicable way of cutting the teeth (by means of end-mills) is comparatively slow, expensive and inaccurate.

CHAPTER III

WORM GEARS

THE tooth action in worm gears (Fig. 8) involves considerably more sliding than occurs in the types of gear already considered, and in the past this caused worm gearing to be regarded with disfavour. Excessive friction led to serious loss of power and to heavy wear of the tooth surfaces. Careful study of contact conditions and the use of appropriate materials have combined to permit the economical use of worm gears in a wide range of practical applications.

If the worm be imagined to be cut by a plane passing

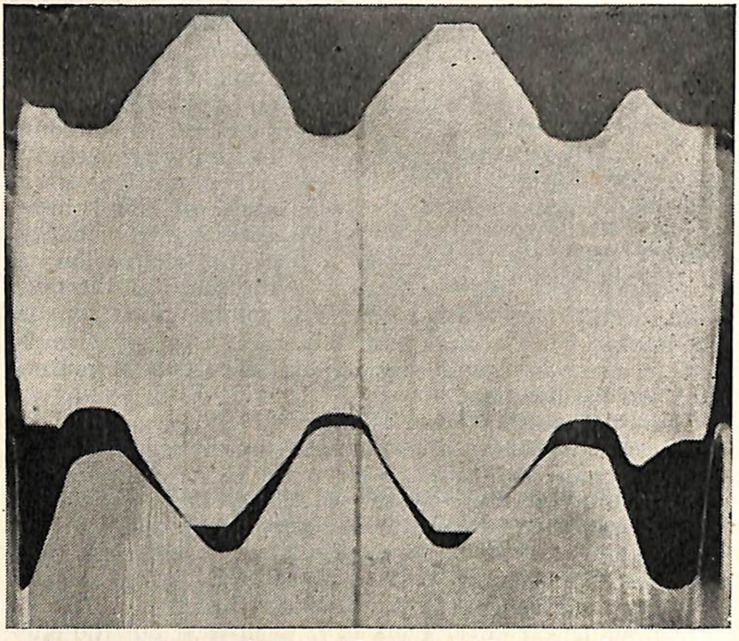


FIG. 22.—Section of worm and worm-wheel on central plane.

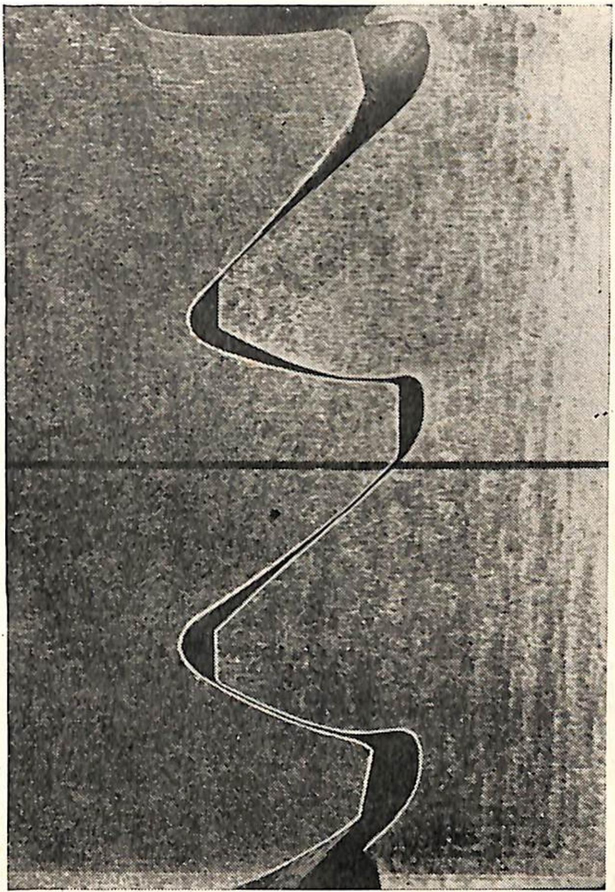


FIG. 23.—Section of worm and worm-wheel on offset plane.

through its axis and lying perpendicular to that of the worm-wheel the threads appear as a series of rack teeth. Rotation of the worm gives these teeth an apparent motion parallel to the axis of the worm, and the form of the worm-wheel teeth examined in the same plane must be such as to mesh accurately with such rack teeth (see Fig. 22).

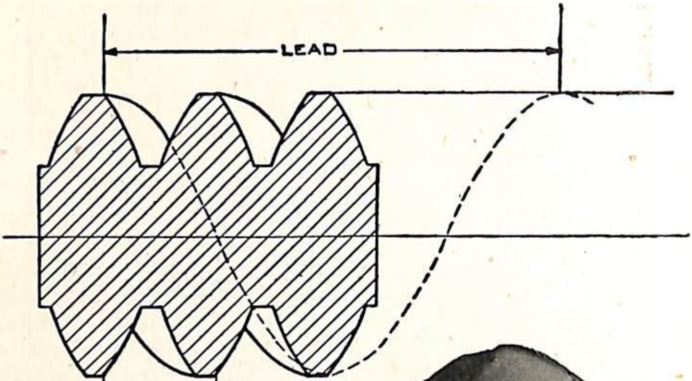


Fig. 25 (*above*).—Dimensions of worm.

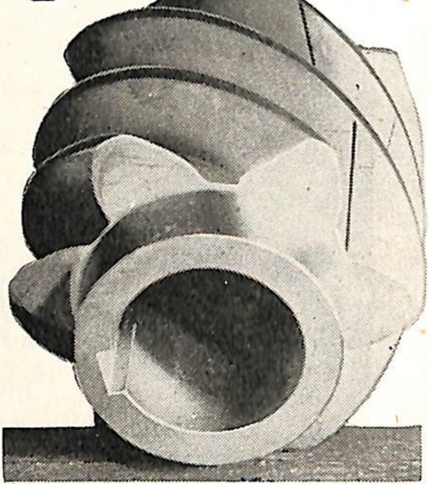


Fig. 24 (*right*). — Straight line on involute helicoidal worm thread.

On planes parallel to this central one, the worm threads show different shapes and the worm-wheel teeth must also differ in form in such planes from that which they have in the central plane (see Fig. 23).

The complex shape of worm-wheel teeth makes it impracticable to produce them by any method other than generation by a cutter of the same form as the mating worm. In this respect the generation of worm-wheel teeth differs from the normal method of producing spur, helical or bevel gear teeth, for mating gears of these types are each generated by a cutter having a tooth form depending only on the normal base pitch required. One cutter can produce gears of any size provided only that the base pitch is specified. A hob for generating worm-wheel teeth can be used only if the mating worm is of the same essential dimensions as the hob.

It is not difficult to choose a worm thread form which will lead to satisfactory worm-wheel tooth shapes in the central plane, but shapes in other planes may be unsuitable unless special care is taken. The British Standard worm thread is of involute form in a plane perpendicular to the axis of the worm, or, in other words, the worm is actually a single-helical involute pinion. The kinematic

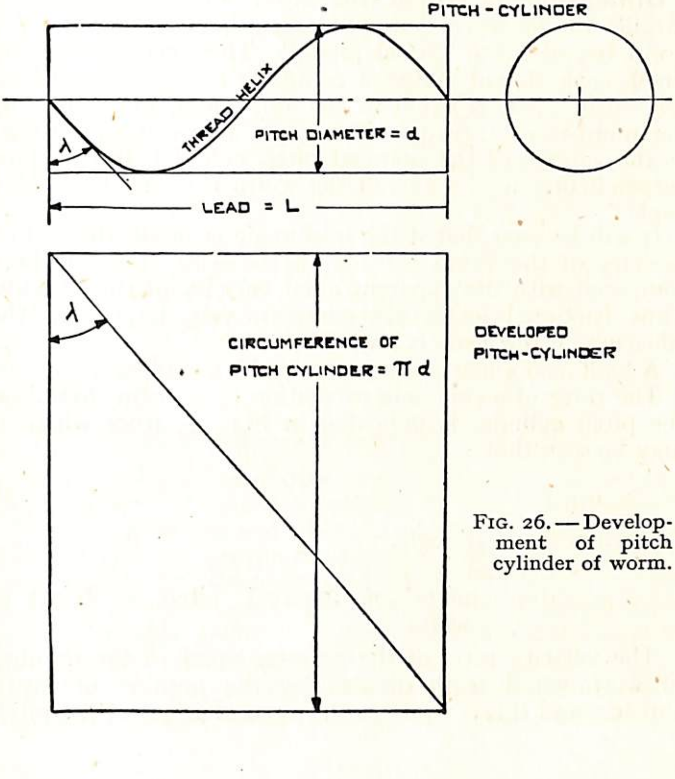

PITCH - CYLINDER
THREAD HELIX
λ
PITCH DIAMETER = d
LEAD = L
λ
CIRCUMFERENCE OF
PITCH CYLINDER = π d
DEVELOPED
PITCH-CYLINDER

Fig. 26.—Development of pitch cylinder of worm.

properties of the involute are not utilised in the worm gear and this thread form is adopted because it offers certain advantages in manufacture and checking for accuracy. The normal pressure angle is 20° except in cases where certain geometrical considerations make a greater pressure angle necessary.

As indicated in Fig. 24, an accurate thread profile of this type contains a straight line and consequently shape errors may be detected by comparison with a straight-edge. No such facility exists in the case of the worm-wheel teeth, and the only practicable method of checking them is by mounting the worm-wheel in correct relation to an accurate worm and observing the nature of the contact between them by the marking on the worm-wheel teeth.

Dimensions of Worm Gears.—The distance measured parallel to the worm axis between adjacent threads of a worm is called the "axial pitch." The axial distance in which each thread makes a complete revolution is called the "lead"; it is equal to the axial pitch multiplied by the number of threads. The angle between the thread at the surface of the nominal pitch cylinder and a plane perpendicular to the axis of the worm is called the "lead angle."

It will be seen that if the lead angle is small, the sliding velocity of the worm threads on the wheel teeth is high compared with the apparent axial velocity of the threads. Thus frictional losses are comparatively large and the efficiency of the gears is low.

A high lead angle is necessary for high efficiency.

The trace of a complete revolution of a worm thread on the pitch cylinder is indicated in Fig. 26, from which it may be seen that

$$\tan \lambda = \frac{L}{\pi d}$$

where λ=lead angle.
L=lead.
d=diameter of nominal pitch cylinder of worm.

The velocity ratio of the gears is equal to the number of worm-wheel teeth divided by the number of worm threads, and this is equal to the circumference of the pitch circle of the worm-wheel divided by the lead of the worm. Hence the lead is fixed if the wheel diameter is fixed, and therefore the highest value of λ is secured by using the smallest worm diameter.

Contact Between Worm and Wheel.—At any instant each worm thread touches one or more worm-wheel teeth along a curved line. As the gears revolve, the various lines of contact change their positions in space in such a way as always to lie on a particular surface called the " zone of contact." The boundaries of the zone of contact are decided by the thread form and by the diameters and widths of the worm and wheel. In some particular angular position of the gears the total length of the lines of contact is a minimum which is used in determining the load capacity of the gears. Consideration of the shape of the zone of contact makes it possible to decide the maximum useful face-widths of worm and wheel.

FIG. 27.—Zone of contact between worm and worm-wheel.

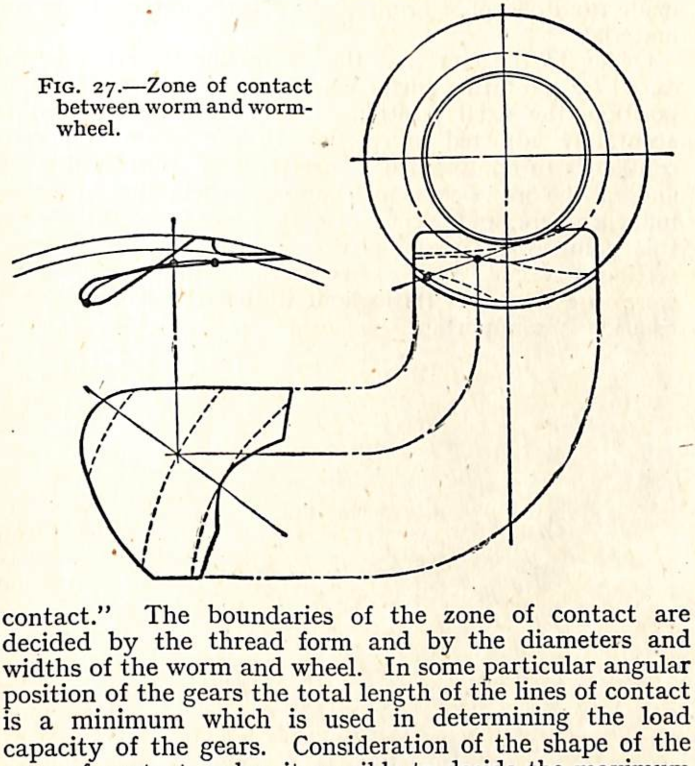


Fig. 27 shows the shape of the zone of contact for a typical worm and worm-wheel.

Each worm-wheel tooth enters the contact zone obliquely, with the result that load is applied to it in a gradual manner and smooth action is easily obtained. Given suitable materials and lubrication, worm gearing is the quietest type of toothed gear. The curvature of the surfaces at any line of contact is less than in spur, helical or bevel gears of the same general dimensions and the better "fitting" of the surfaces leads to substantial load-capacity despite the fact that the worm-wheel is usually made from phosphor bronze, which is a comparatively soft material.

Owing to the fact that the worm-wheel teeth are produced by a rotating cutter whose axis does not change its position, the axial position of the worm-wheel must be accurately adjusted in relation to the worm if correct contact is to be obtained. Distortion of worm and wheel and of the members which support them has a serious influence here, and special attention has to be directed to this point when worm gears are being mounted before setting to work. As the cross-sectional dimensions of the worm are the same throughout its length, accurate axial location is unimportant.

CHAPTER IV

Gear Generation

Sunderland Process for Spur Gears.—The moulding generation process depicted in Figs. 12 and 13 appears in the Sunderland spur gear generator, with the single modification that the rack is made with sharp cutting edges and is reciprocated parallel to the axis of the blank so that the teeth cut their way into the metal instead of being pushed into the plastic blank in the manner already described.

In the machine shown in Fig. 28 the cutter-box moves horizontally to and fro across the whole width of the

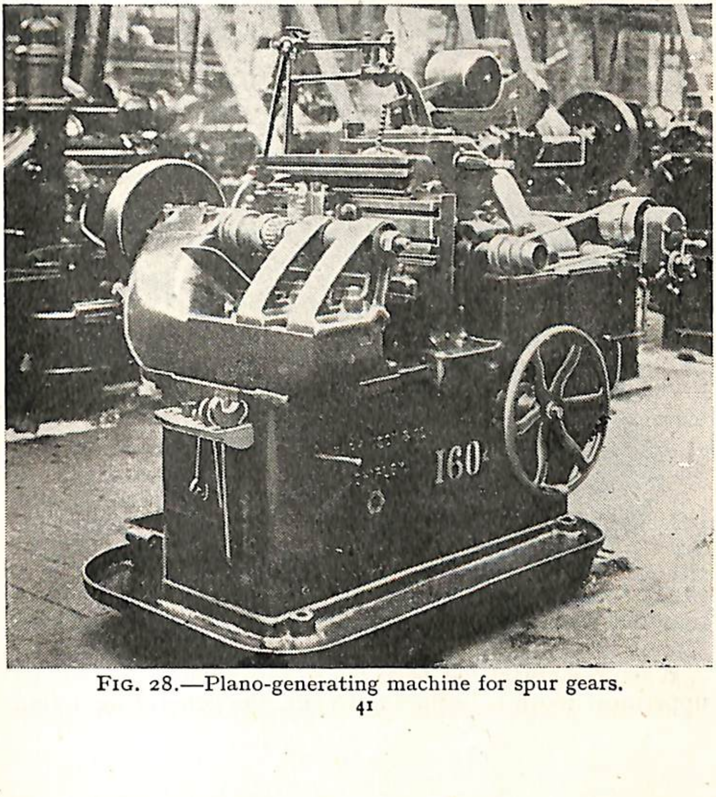


Fig. 28.—Plano-generating machine for spur gears.

blank, and the slide which guides it is given a slow vertical movement in unison with rotation of the blank. Whilst the cutter moves downwards through a distance equal to the pitch of its teeth the blank turns through an angle

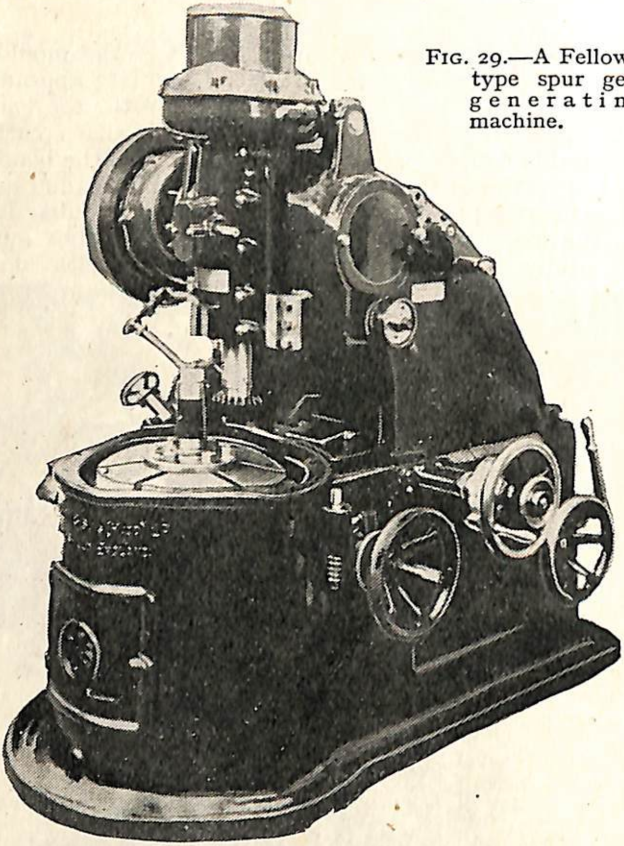


FIG. 29.—A Fellows-type spur gear generating machine.

equal to one revolution divided by the number of teeth required in the work, the " index change gears " and the " pitch change gears " being selected to secure this result.

When the cutter has been fed so far downwards that its uppermost tooth is beginning to cut, the generating action

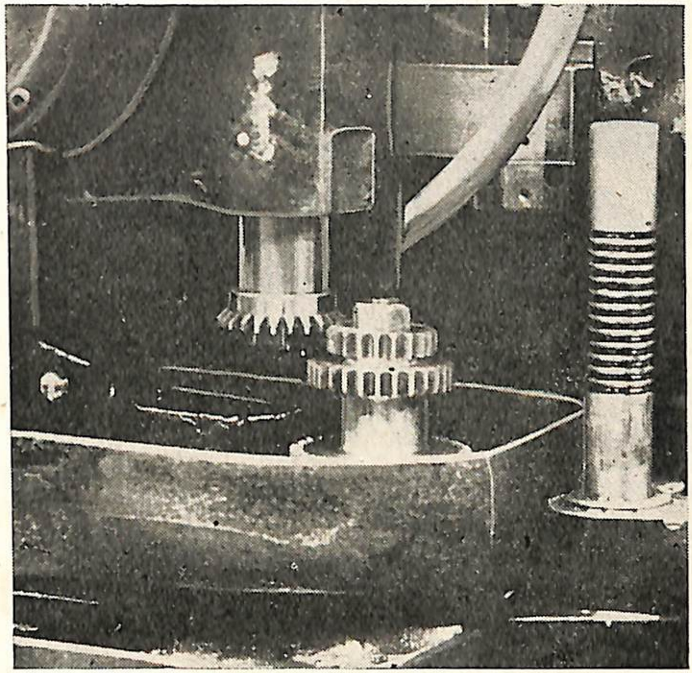


FIG. 30.—A Fellows-type machine cutting gears.

is automatically stopped. The cutter is arrested at one end of its cutting stroke, clear of the blank, and is moved upwards through a distance equal to a whole number of pitches without rotation of the blank. Its feed mechanism is then reconnected to that which rotates the blank, a short generating movement is given to cutter and blank to take up backlash, and the cutting strokes and " rolling " motions are resumed. This cycle is automatically repeated until the entire circumference of the blank has been treated.

If the pitch of the teeth is small compared with the capacity of the machine, the full depth of tooth may be cut at once, and complete generation of all the teeth effected in one revolution of the blank. In general, however, two or more revolutions are necessary, the cutter being reset after each revolution to sink more deeply into the blank.

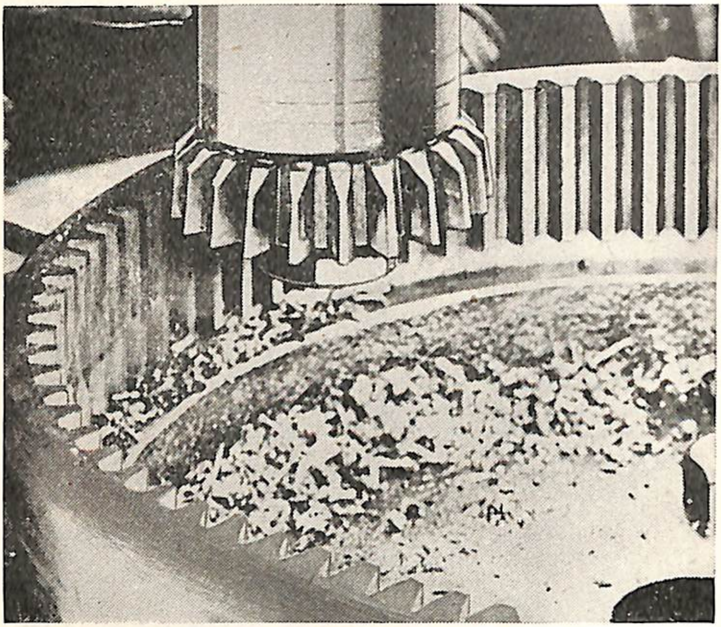


FIG. 31.—A Fellows-type machine cutting an internal spur gear.

In some circumstances this type of machine may use a special cutter-box mounting, two cutters facing in opposite directions so that cutting takes place in both directions of motion of the cutter-box across the face of the gear blank.

The cutter is sharpened by grinding its front face, the form of the cutting edges being unaffected.

Fellows Process for Spur Gears.—The Fellows process resembles the Sunderland process in that it uses a cutter which reciprocates parallel to the axis of the gear, but differs from it in the form of the cutter. The effective form of the Fellows cutter is that of a spur gear, the teeth being tapered to give cutting relief on the profile and the tip.

The necessary generation motion of cutter and blank is secured by continuous rotation of each of them at rates inversely proportional to their numbers of teeth, and there is nothing to correspond to the periodic resetting of the cutter that occurs in the Sunderland process.

To avoid rubbing of the edges of the cutter on its idle return strokes the work is drawn a short distance away from it at the end of each cutting stroke and replaced at the beginning of the next one.

The Fellows process can be applied to a gear which is situated close to a shoulder and it can, therefore, handle cluster gears (Fig. 30). The Sunderland process suffers some restriction in this direction (unless a specially shaped cutter is used) because part of the cutter-box must be in front of the cutter.

The Fellows process can be used to generate internal gears (Fig. 31), and except for the Sykes process, which differs from it in detail, is the only true generating process applicable to that type of gear.

The Hobbing Process for Spur Gears.—The most accurate spur gear generating process is that based on the use of the cutting tool known as the "hob" (Fig. 32).

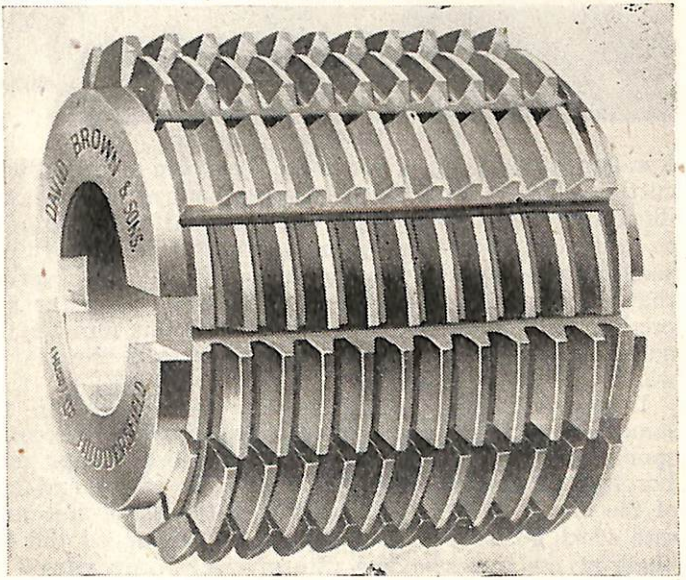


FIG. 32.—A hob for cutting spur or helical gears.

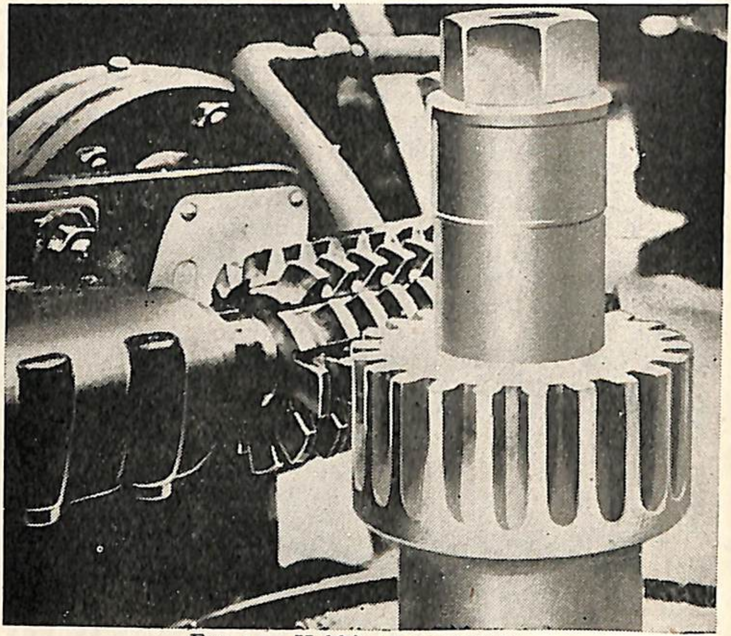


FIG. 33.—Hobbing a spur gear.

Fundamentally this is a spiral gear or worm provided with cutting edges. Very often—as in the case of the one illustrated—the hob has only one thread (if it be regarded as equivalent to a worm) or one tooth (if it be looked upon as a spiral gear). In this connection it may be observed that the provision of "gashes" (or "flutes") in the hob produces a number of cutting teeth, but this number has no essential relation to the number of teeth in the gear to which the hob corresponds.

In the hobbing machine the hob is mounted in the same relation to the blank to be cut as would be the corresponding spiral gear to mesh with the finished spur gear. For starting the cut, the hob is set clear of the end face of the blank, and the two are then rotated, the hob at a rate which gives its edges a suitable cutting speed and the blank at the corresponding rate inversely proportional to its number of teeth. The hob is slowly fed parallel to the axis of the blank until it has traversed the whole face-width. If the pitch of the gear is small enough, the hob may be set to the full depth of tooth, and one traverse suffices to complete the gear; usually, however, two or more cuts are required.

The fundamental accuracy of the hobbing process is associated with its continuous nature. For each cut the whole width and circumference of the blank are covered without change in speed in any part of the hobbing machine, and there is thus no possibility of introduction of error by variation of stress at any point.

The hobbing process applied to the generation of spur gear teeth is shown in progress in Fig. 33. The cutting of the teeth has commenced at the upper edge of the blank and part of the face-width has been covered. The bottom surfaces of the grooves cut by the hob teeth slope gradually to the outer surface of the blank, this " run-out " resulting from the circular path of the tips of the hob teeth. When the hob has been moved vertically downwards until full-depth teeth have been cut throughout the width of the

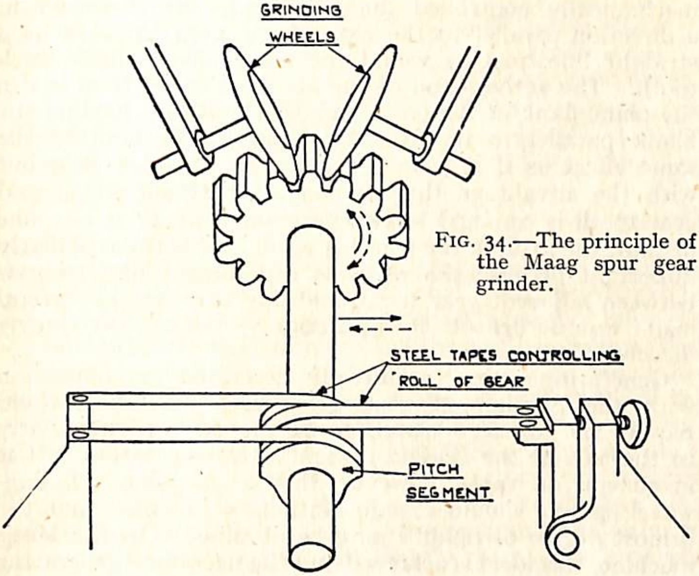


FIG. 34.—The principle of the Maag spur gear grinder.

blank, the run-out of the hob teeth demands clearance which has to be taken into account when setting-up the blank on the hobbing machine. Also, in the case of a pinion made solid with its shaft, this consideration limits the diameter of collars situated near to the ends of the gear teeth.

Grinding of Spur Gear Teeth.—To secure the greatest load capacity in spur gears, it is necessary to make them of case-hardened steel, and if a high degree of accuracy is demanded, the profiles must be finished by a grinding process in order to remove the inaccuracies resulting from distortion in hardening. Several types of such grinding machines operate on the generation principle, and a typical one is the Maag, illustrated in Fig. 34.

The grinding of the teeth is done by the outer faces of the abrasive wheels, each of which is set to correspond to one flank of a tooth of the imaginary basic rack. The abrasive wheel is moulded to a saucer-shape, contact with the gear tooth being limited to a narrow band near the edge of the outer face. This band is trimmed by a mechanically controlled diamond and, when viewed in a direction parallel to the axis of the gear, appears as a straight line, just as would the flank of the basic rack tooth. The active band of the abrasive wheel thus lies in the plane flank of the basic rack tooth and by feeding the blank parallel to its axis the band has ultimately the same effect as if it were a continuous abrasive face but with the advantage that as contact between wheel and gear tooth is confined to two very small areas at any one instant, the load on the wheel is small. This is particularly important because the wheel is necessarily thin, to pass between adjacent gear teeth, and any considerable lateral load would deflect it appreciably, thus introducing inaccuracy.

Generating action, as already described in connection with the Sunderland gear generating machine, would require the abrasive wheels to be moved perpendicularly to the axis of the gear in unison with its rotation. It is in general desirable, however, that a precision grinding-wheel spindle should remain stationary in order that the utmost degree of rigidity may be obtained. In the Maag machine, this ideal is achieved and the necessary generating action is secured by giving the blank both straight-line motion perpendicular to its axis and rotation about that axis at the correct relative rate.

Obtaining the Double Motion.—This double motion is obtained, simply and accurately, by means of the segment and tapes shown in Fig. 34. The radius of the segment is that of the desired pitch circle of the gear teeth. The segment is mounted on the end of the work spindle into which the work arbor is fitted and four steel tapes are attached to it, two passing horizontally to a fixed bracket on one side of the machine, the other two lying between them and extending to a tensioning lever on the other side of the machine. (The tapes are duplicated as a safeguard against damage should one of them break.) Thus when the work head moves perpendicularly to the axis of the work spindle, the tapes cause the spindle to rotate as if the segment were rolling without slip on a flat surface. Hence the gear teeth have the same rolling action, and contact with the appropriately positioned abrasive wheels thus produces ground involute profiles.

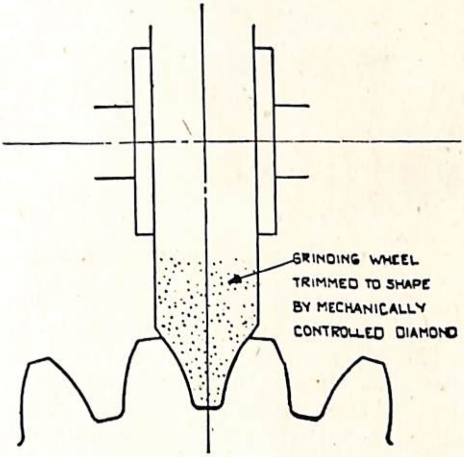


Fig. 35.—Formed-wheel grinding of a spur gear.

Generating action is produced by lateral reciprocation of the work head, and to cover the whole face-width of the teeth concerned, the gear is slowly fed parallel to its axis. After it has moved clear of the abrasive wheels an automatic indexing motion occurs and on the next traverse two other tooth profiles are treated. When dealing with gears of large pitch the abrasive wheels may be situated in the same tooth space, but usually they are in adjacent

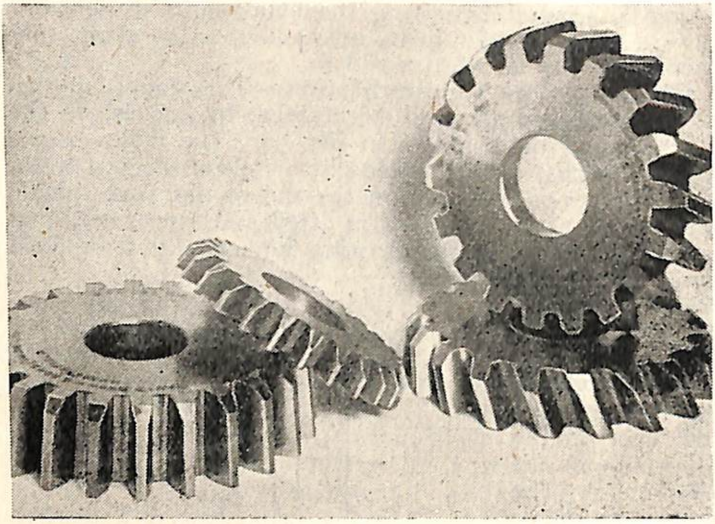


FIG. 36.—Spur and helical Fellows-type cutters.

spaces and when grinding gears of very small pitch they may have to be even more widely separated.

The Maag machine incorporates special mechanism to offset inaccuracy that would otherwise result from wear of the abrasive wheels. A diamond-faced feeler periodically follows a path in which it should make contact with the active band of the abrasive wheel. If it fails to do so by reason of wear of the wheel, the position of the wheel is automatically adjusted until contact is established. By this means the effective face of the abrasive wheel is kept within one-quarter of a thousandth of an inch of its correct position, despite wear.

The Formed-Wheel Process.—Certain types of spur-gear tooth grinding machines do not use the generating principle but depend upon direct reproduction of the shape of the abrasive wheel in the gear teeth. Machines of this nature are described as operating by the "formed-wheel process." The wheel is trimmed by diamonds under the control of mechanism whose movements depend on the guidance of plates cut to the shape of the required tooth profile enlarged about five times. It is usual to grind the two profiles and the root surface of each tooth space at once, and this makes the form-grinding process more rapid than generation grinding, where the abrasive wheels are less effectively used. (See Fig. 35.) The gear is moved parallel to its axis so that the abrasive wheel may grind the whole face-width; after that, rotation of the gear through an appropriate angle causes the wheel to grind a different tooth space on the next traverse.

An advantage of the formed-wheel process is that it facilitates slight modifications of tooth profile such as are sometimes found necessary in heavily loaded gears.

Generation of Helical Gear Teeth.—The spur gear

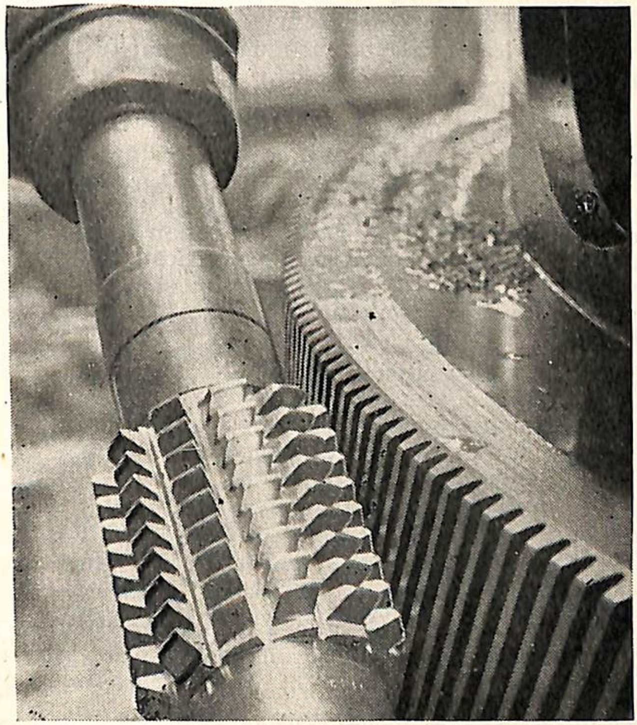


FIG. 37.—Hobbing a helical gear.

generating processes already described are all adaptable, with some complication in mechanism, to the generation of helical gear teeth. For example, in an elaborated form of the machine shown in Fig. 28 the cutter-slide can be set at any desired angle to the axis of the gear and then helical teeth are generated with involute profiles on transverse sections.

A similar adaptation is possible in the case of the Fellows process (Fig. 29). A helical guide has to be provided so that vertical movement of the cutter gives it proportional angular movement, and the cutter itself must have helical teeth. Fig. 36 shows a group of Fellows-type cutters, some for producing straight tooth gears, the others for helical gears. The helix angle of the cutter (and therefore of the gear produced) depends on its diameter, because the lead of the cutter must be that of the helical guide.

Hobbed Helical Gears.—Adaptation of the hobbing process to the generation of helical gear teeth is comparatively easy. The angular position of the hob spindle is adjusted so that the teeth, when cutting, are moving in the direction of the tooth spirals of the gear to be generated (see Fig. 37). Feed of the hob parallel to the axis of the blank introduces a complication because it must be accompanied by rotation of the blank through an angle inversely proportional to the lead of its teeth. Thus if

T=number of teeth in gear
L=lead of gear teeth (in.)
t=number of threads in hob
f=feed of hob (in.) per revolution of hob
N=rotational speed of gear
n=rotational speed of hob

then if there were no feed of hob

$$N=\frac{nt}{T}$$

Since feed of the hob through a distance equal to the lead of the gear would require one additional revolution of the gear, the additional speed of the gear corresponding to feed of the hob at nf inches per minute is $\frac{nf}{L}$ r.p.m.

Hence the speed of the gear is

$$N=\frac{nt}{T}\pm\frac{nf}{L}=n\left(\frac{t}{T}\pm\frac{f}{L}\right)$$

r.p.m.

and this relation determines the ratio of the change-gears required in setting up the machine. The double sign is used because, according to the relation of direction of feed of hob to hand of gear teeth and direction of rotation it may be necessary in one case to use + and in another to use the − sign.

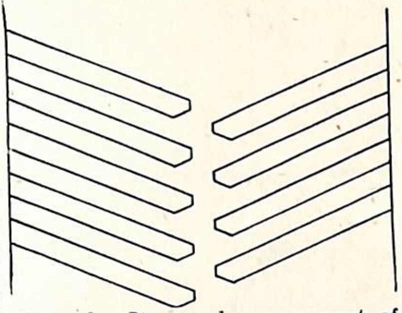


FIG. 38.—Staggered arrangement of teeth in a hobbed double-helical gear.

By adding a differential gear to the mechanism of the gear-cutting machine it is possible to change the rate of feed or even to discontinue it altogether without stopping the rotation of hob and blank. This is often a great convenience in practice, but the additional complication in the machine tends to have an adverse effect on the accuracy of the product.

The hobbing process is widely used for the production of double-helical gears, and the "run-out" of the hob demands a gap between the right- and left-hand helices. By adopting staggered teeth and permitting a chamfering of the ends of the teeth the loss of effective face-width at the centre is minimised (see Fig. 38).

Plano-Generating Processes for Double-Helical Gear Teeth.—The Sunderland process, with rack-shaped cutters, may be used for double-helical gears, but it involves a special type of machine of which an example is shown in Fig. 39. Here the cutter-boxes are guided to move in directions oppositely inclined to the axis of the blank, but the cutters themselves are set perpendicular to it. One cutter moves towards the centre of the face-width as the other recedes from it; cutting occurs on the "in" strokes only. By setting the cutters so that they very slightly overrun the central plane, each cuts a clearance for the other, and it thus becomes possible to

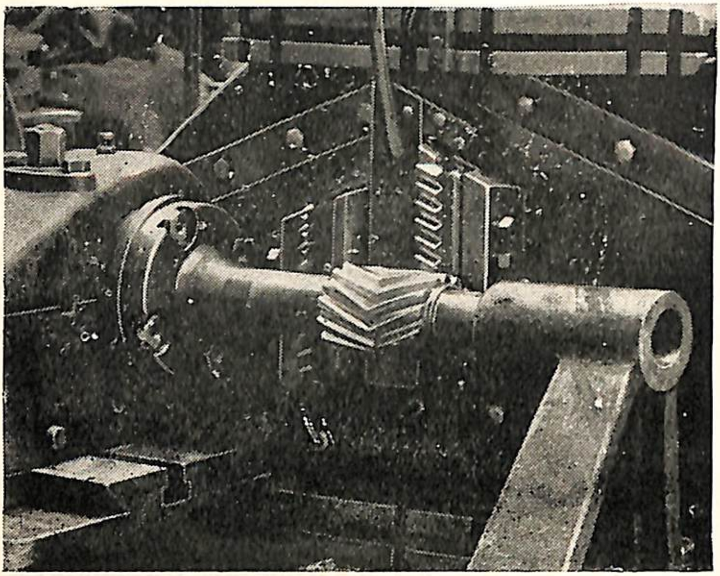


FIG. 39.—Plano-generating machine for double-helical gears.

produce *continuous* double-helical teeth by a generating process. This type of tooth is somewhat stronger than the double-helical tooth with a central gap, but the advantage is slight in gears of normal proportions. Furthermore, the load capacity of few double-helical gears is limited by tooth strength.

The Sykes System.—The Sykes process for producing double helical gears is the counterpart of the Fellows process; it uses simultaneously two helical pinion-type cutters, one of each hand. The mechanical construction of Sykes machines is elaborate, inasmuch as there is a spiral guide for each cutter, a relieving motion for each cutter to avoid rubbing on the idle stroke, and a hollow-shaft mounting for one cutter encircling the shaft which drives the other cutter. Nevertheless, machines of this type can cut accurate gears at high rates of production, and this is a tribute to care in detail design and to excellence of manufacture.

Profile-Grinding of Helical Gears.—The Maag, and similar generation-grinding processes for spur gears, may be adapted to profile-grinding of helical gears at the cost, of course, of some complication in machinery. The formed-wheel process may be similarly adapted, but a difficulty is that the normal section of the abrasive wheel is not exactly reproduced in the gear owing to interference between the wheel and the helical teeth. Whilst it is possible to determine by calculation the shape of an abrasive wheel necessary to grind a helical tooth which is of involute form on the transverse section, the operation is more involved than in the case of spur gears, and the formed-wheel process has had comparatively little application to the grinding of helical gear teeth.

Generation of Straight Bevel Gear Teeth.—The fundamental principle underlying the operation of most bevel gear generating machines is that of "rolling" the

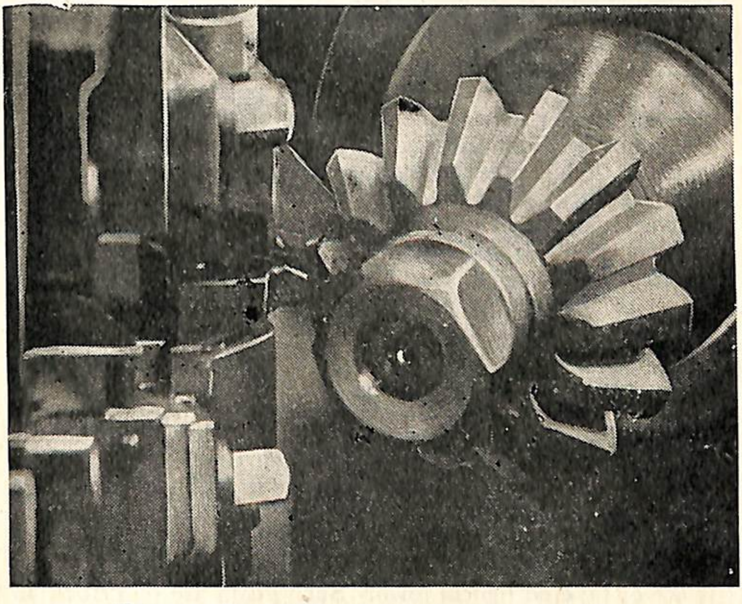


FIG. 40.—Straight bevel gear generating machine.

blank with a crown-wheel which is imaginary save for a cutter blade or blades continually sweeping two of its tooth flanks.

In Fig. 40 two straight-sided cutter blades can be distinguished. These are reciprocated along lines converging at the apex of the machine and the cutter head which carries their guides is given partial rotation about the imaginary crown-wheel's axis, which passes through the apex. Simultaneously the blank (whose axis also passes through the apex) is given proportional rotation so that blank and imaginary crown-wheel are correctly " rolled " together and the reciprocating cutter blades then form teeth which will mesh correctly with teeth in other gears generated in the same way.

After the rolling has proceeded so far that the tooth flanks concerned are fully generated, the blank is withdrawn from the cutters by rotation of the work head about a vertical axis passing through the apex, and the blank is rotated through an angle corresponding to one tooth pitch. At the same time cutter head and blank are rotated back to the initial generating position. The blank is then returned to the cutting position, and generation of another pair of tooth profiles begins. These operations are entirely automatic and the cycle is repeated until all the gear teeth have been generated.

The generation of the teeth of the mating gear proceeds in exactly the same way except, of course, that the machine is " set up " to suit the different pitch angle and the different number of teeth. Thus the two gears are generated so as to mesh with a common imaginary crown-wheel, and as the teeth of such a wheel are symmetrical (neglecting clearance) about the pitch plane, the two gears will mesh correctly with each other.

Generation of Single-Helical Bevel Gears.—The process just described may be applied with slight modification to the production of bevel gears with helical teeth. The path of each cutter blade is still a straight line which, however, does not intersect the axis of the imaginary crown-wheel. The teeth of that wheel are thus straight but not radial, and the teeth of bevel gears generated from it are of tapered helical form. Such gears are described as " single-helical bevel " gears.

Generation of Spiral Bevel Gears.—Consideration of tooth action in helical bevel gears does not demand that the teeth of the imaginary crown-wheel be straight. The advantages of helical tooth formation are secured if the crown-wheel teeth are of almost any form other than the straight radial one that characterises the straight bevel gear. A bevel gear whose corresponding crown-wheel teeth are not straight is called a " spiral bevel " gear.

The Gleason spiral bevel gear is based on a crown-wheel whose teeth are of circular arc form (Fig. 41). The advantage of this is that the cutter blade may be carried by a revolving disc and may thus return to take further cuts by virtue of successive rotations of the disc. In practice the disc is fitted with a number of equally spaced cutter blades, so that cuts on any one tooth flank take place in quick succession and cutting proceeds more smoothly than is the case in a machine having reciprocating cutters. Each cutter blade acts on one tooth

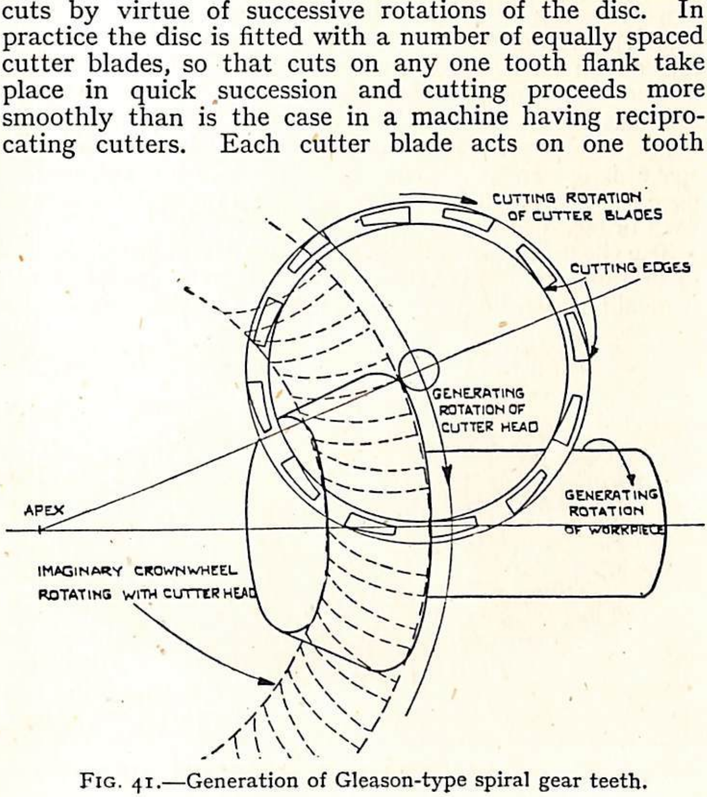

CUTTING ROTATION OF CUTTER BLADES
CUTTING EDGES
GENERATING ROTATION OF CUTTER HEAD
GENERATING ROTATION OF WORKPIECE
APEX
IMAGINARY CROWNWHEEL ROTATING WITH CUTTER HEAD

FIG. 41.—Generation of Gleason-type spiral gear teeth.

flank only, and successive blades deal with opposite flanks. This feature can be distinguished in Fig. 42, which shows a typical Gleason spiral bevel cutter having "inside" and "outside" blades placed alternately along the edge of the circular body. Fig. 43 shows a Gleason spiral bevel gear generating machine operating on a wheel, and Fig. 44 shows a similar machine generating teeth in a small pinion.

The rotation of the cutter about its axis causes its blades to sweep out one tooth space of the crown-wheel. Simultaneously the cutter is moved bodily in a manner corresponding to a partial rotation of the imaginary crown-wheel and the gear blank is given the appropriate rotation in unison with it. These movements continue far enough for complete generation of the flanks adjacent to one tooth space. Then the cutter is withdrawn from the blank, the "rolling" motions are reversed and the blank "indexed" so that on the next cycle another tooth space is generated. The machine is fully automatic; once set to work it generates all the teeth in the gear and then brings itself to rest.

The Gleason process is the most widely employed method of producing spiral bevel gears, and it finds special favour in dealing with large quantities of the small gears of this

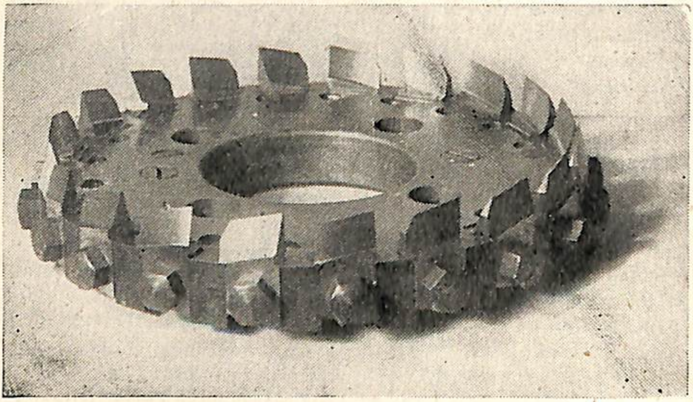


FIG. 42.—A Gleason-type spiral bevel gear cutter

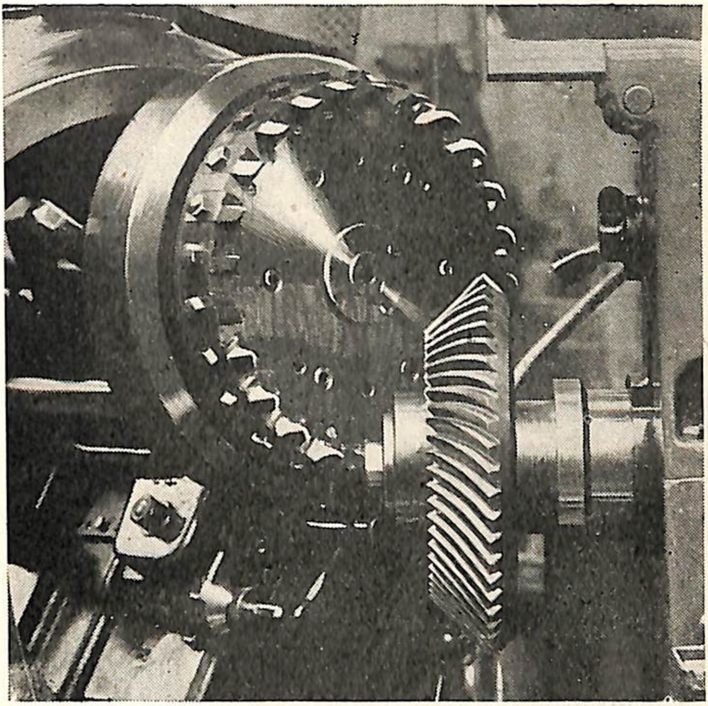


FIG. 43.—Gleason-type spiral bevel gear generating machine.

type used in automobile rear axles. By certain modifications in the machine it can be arranged to produce " offset spiral bevel " gears to which the name " hypoid " has been applied. An example of this type of gear is shown in Fig. 45.

Another modification of the Gleason spiral bevel gear is the " formate " gear. In this the teeth of the wheel are not generated at all, the blank and cutter head being held stationary whilst each tooth space is treated and the teeth, therefore, have the same shape as the cutter blade. The pinion teeth are generated substantially in the standard Gleason manner. This process is a specialised one and can be applied economically only when several

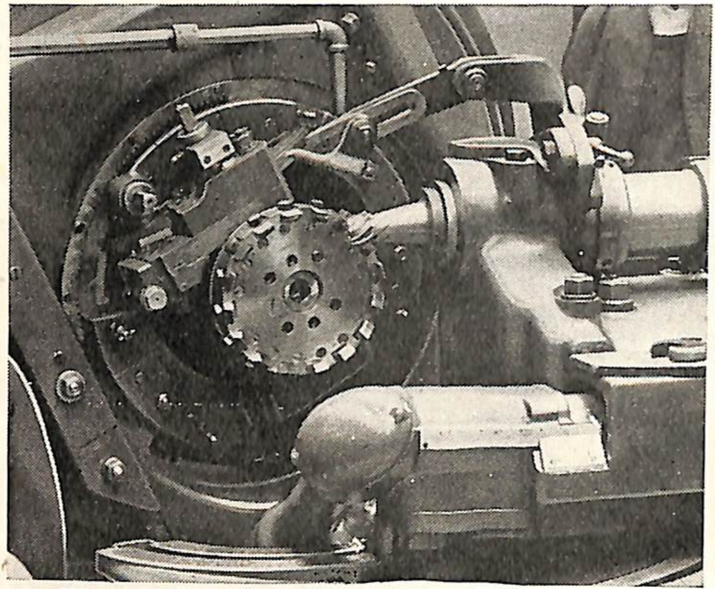


FIG. 44.—Generating a Gleason-type spiral bevel pinion.

hundreds of pairs of gears are to be made in one batch.

Worm-Wheels.—The only practicable way of producing accurate worm-wheel teeth is by generation with a hob whose essential dimensions are the same as those of the worm which is to mate with the wheel. A typical worm-wheel generator is shown in Fig. 46. The hob is mounted horizontally and rotates at a rate which imparts a suitable cutting speed to its teeth. The worm-wheel blank is arranged with its axis vertical, and its speed of rotation is that which the completed worm-wheel would have when meshed with a worm running at the hob's speed. The head carrying the hob and its driving mechanism is able to slide horizontally, and during the generating process it is caused to

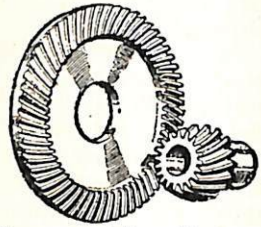


FIG. 45.—A hypoid gear.

do this so that the hob penetrates more and more deeply into the blank until finally the distance between their axes is equal to the centre distance of the finished gears.

A hob is an expensive tool to manufacture, and its cost may be prohibitive if it is required to produce only one or two worm-wheels. In such cases it is common to use a "fly-hob," which corresponds to one tooth of the complete hob (see Fig. 47). In order that this tool may effect complete generation of the worm-wheel, the two must have the appropriate relative speeds as for worm and worm-wheel, and in addition the hob must be fed along its own axis so that it gradually penetrates more deeply into the blank, the centre distance between them remaining constant throughout the generating operation. This

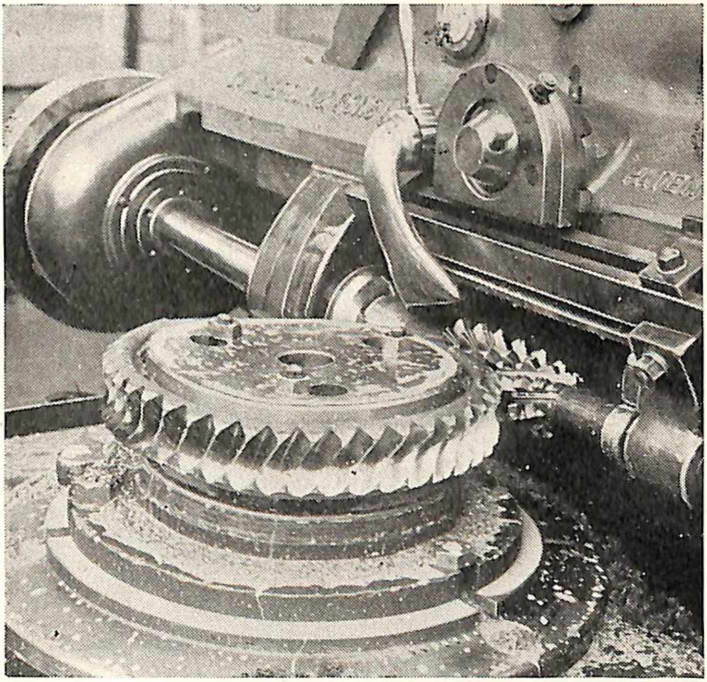


FIG. 46.—A worm-wheel generator.

axial feed of the hob means that the worm-wheel blank must be given the corresponding rotation superimposed on the one which it already has in association with rotation of the hob.

Generation by "fly-hobbing" is naturally much slower than is possible with a complete hob, because the single tooth has to do the amount of work that is spread over perhaps fifteen teeth in the complete hob. To obtain smoothly finished teeth the axial feed of the hob has to be slow.

Axial feed is often adopted when using complete hobs with the object of securing a higher degree of tooth finish than is practicable with the direct "in-feed." For such applications the hob is usually made of tapered form over part of its length, the roughing-out of the worm-wheel teeth being effected by the teeth at the small end of the hob.

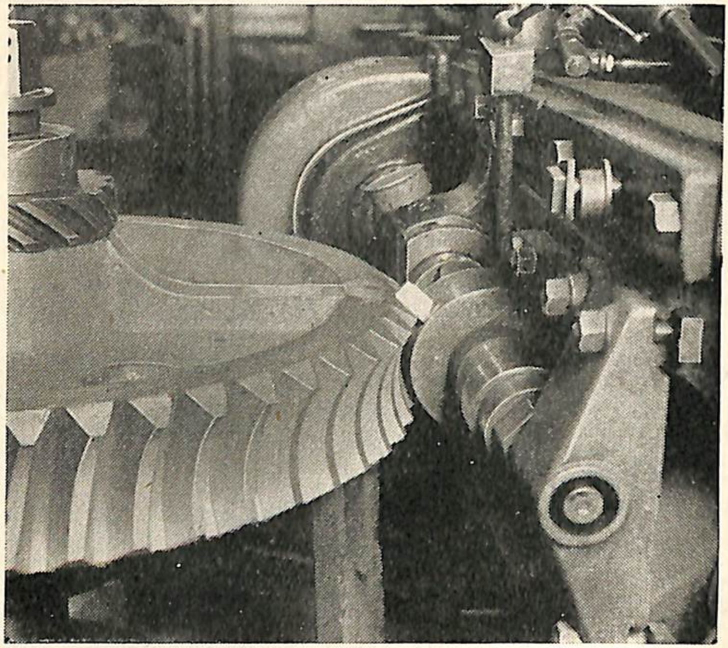


FIG. 47.—The generation of a worm-wheel by means of a "fly-hob."

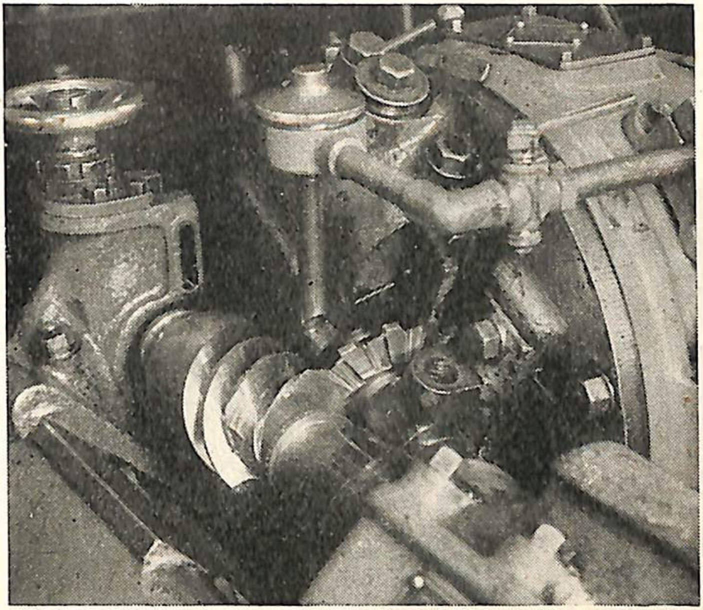


FIG. 48.—Cutting a worm in a thread-milling machine.

When quantities of worm-wheels are being generated it is usual to finish the cutting of the teeth in a separate operation from the initial rough-cutting. In such cases the finishing of the teeth is often carried out by means of a "serrated hob" which has a large number of finely-pitched teeth.

Cutting of Worm Threads.—The worm threads are cut in a thread-milling machine (Fig. 48) or (in certain circumstances) by hobbing. The thread-milling cutter rotates about an axis whose angular position is adjustable to suit the lead-angle of the worm to be produced. The table which carries the work spindle can slide parallel to the spindle and in actual operation this sliding occurs in unison with rotation of the blank, change-gears being provided in order to afford means of accommodating any required lead of worm. This combination of rotation and axial movement of the blank past the rotating cutter causes it to cut a helical groove in the blank. When the whole width of the blank has been covered, the blank is quickly returned to the starting-point, and after "indexing," the process is repeated to cut another thread. Further repetition takes place until all the threads have been completed.

Grinding Worms.—Worm threads are nearly always subjected to a grinding operation, and this is carried out on the type of machine shown in Fig. 49. The movements of the machine are similar to those of the thread-miller, but the milling cutter is replaced by an abrasive wheel and the operation is entirely automatic. Each flank of each thread has usually to be ground several times to bring the thread thickness down to the required figure.

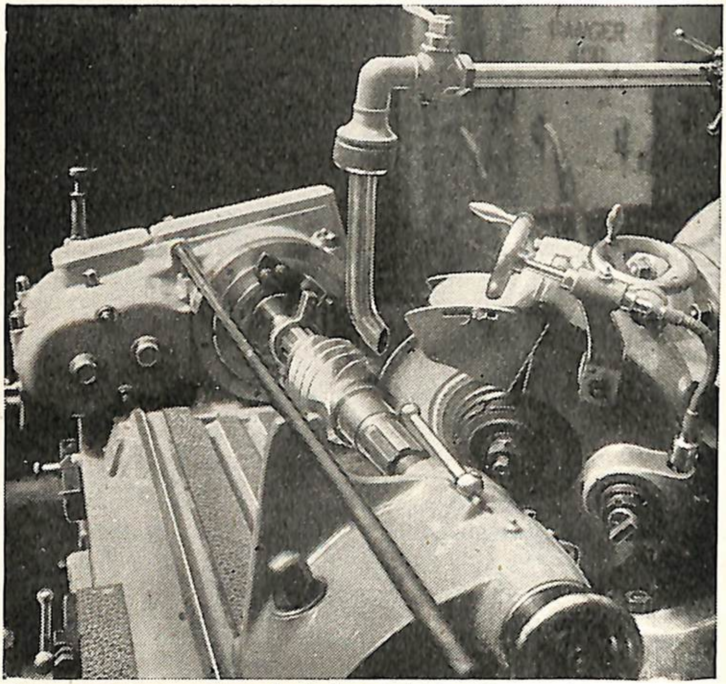


FIG. 49.—A worm-thread grinder, finishing the rough-cut worm.

In grinding worm threads of the involute helicoid type (the British Standard) the abrasive wheel may be flat-sided and this gives the advantage that the wheel may be trimmed by a diamond moving in a straight line. By setting the axis of the abrasive wheel at the appropriate angle it touches the worm thread along the straight line which characterises the involute helicoid worm. On this basis only one thread flank can be ground at once.

Alternatively, two flanks may be ground at the same time, but in this case the abrasive wheel has to be shaped to two curved contours by diamonds moving under the control of specially shaped former plates. The shape of the plates is determined by calculation according to the particular thread form required.

Finishing Worms.—In order to obtain the necessary mirror finish on worm threads the grinding operation must be followed by treatment with fine abrasive. To effect this the worm is rotated in mesh with a lap in the form of a wooden worm-wheel charged with abrasive paste. Power is applied to the worm which thus rotates the worm-wheel against the resistance of an adjustable brake and is consequently polished by the paste. To cover the whole face-width of the worm it is reciprocated in the direction parallel to its axis.

CHAPTER V

GEAR-WHEEL FORMS

SMALL gears may be made in the form of solid discs, but as this is not economical of material the armed wheel is common in large sizes. An example is the cast-steel wheel shown in Fig. 2, and it will be noticed that the arms are of H-section to give adequate strength with a comparatively small amount of material. The particular gear shown has a "split boss," this form of construction being adopted in large gears in order to reduce the strains caused by contraction of the metal as it cools in the mould when in the solid state immediately after casting. When the boss is made in one piece the arms in the finished wheel are in a state of tension; splitting of the boss avoids this. After the boss has been rough-machined, the splits are filled with molten brass, and after this has cooled the boss is consolidated with two steel rings, one shrunk on to each projecting end.

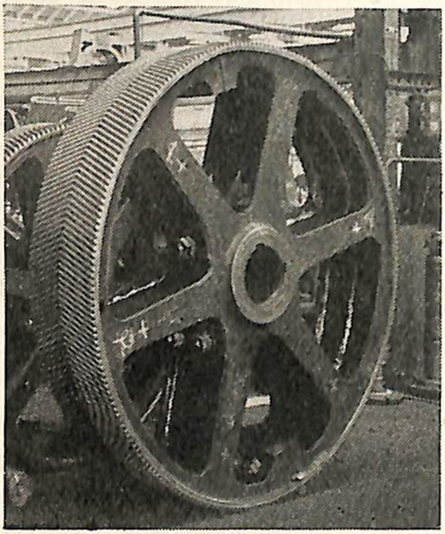


FIG. 50.—Cast-steel double-helical gear-wheel in halves.

An alternative way of diminishing contraction strains is to make the wheel in halves, and although this is more expensive by reason of extra material and machining, the reduction in dimensions of the largest piece to be handled often brings important advantages in transport and

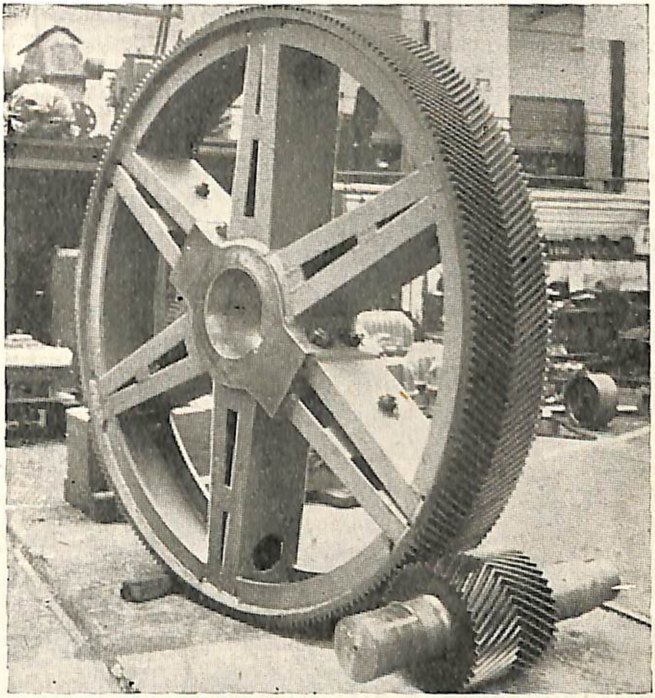


FIG. 51.—A double-helical gear-wheel built up by welding.

erection. Fig. 50 shows a large gear-wheel made in halves and fastened together by means of bolts.

Built-up Gears.—When loading conditions demand the use of a material of such hardness that it cannot be obtained as a casting, the wheel may consist of a forged steel rim shrunk on to a "centre" made from cast-iron which is, of course, comparatively inexpensive.

There is nowadays a tendency to build up steel gear-wheels by welding the rim to arms cut out of plates, and very often the design is such that the wheel is not easily distinguishable from a cast wheel of conventional design. Alternatively the arms may be made from standard rolled-steel sections (see Fig. 51).

Because of the high cost of the phosphor bronze used for worm-wheel teeth, such wheels are usually made as rims secured to cast-iron centres by shrinking and pegging, by bolting, or by welding.

Pinions.—The pinion of a high-ratio pair has often to be made solid with its shaft because a bore large enough to accommodate a separate shaft of adequate strength would leave insufficient metal below the roots of the teeth. In some cases (*e.g.*, Fig. 52) the effective face-width necessary for load capacity is so great in relation to the diameter of the pinion that extra support must be provided by a third bearing separating the two halves of the face-width.

Automobile Gears.—Weight considerations demand

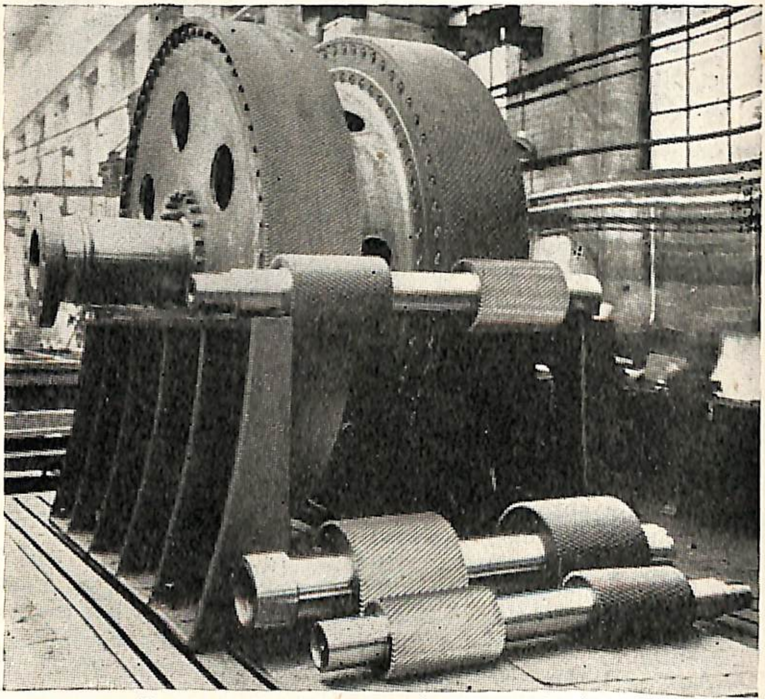


FIG. 52.—Turbine reduction gears, showing each pinion arranged for a third bearing between the right-hand and left-hand helices.

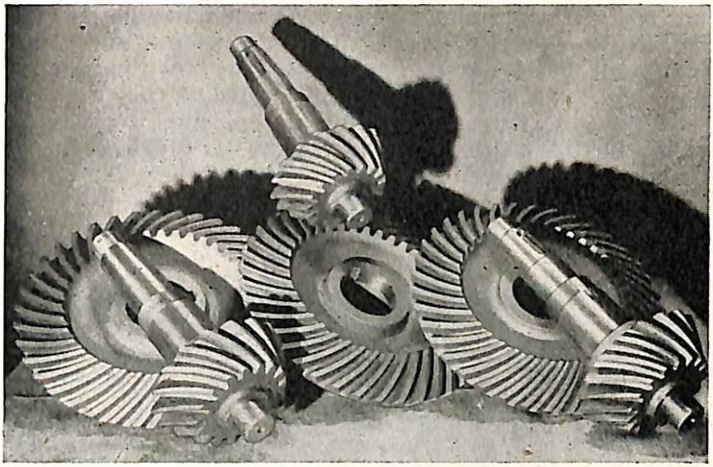


FIG. 53.—Spiral bevel gears for automobile rear-axle drives.

the minimum dimensions in automobile gears, and case-hardened steel is almost invariably used in such cases because of its high load capacity. Fig. 53 shows a typical group of rear-axle spiral bevel gears, the pinion being made solid with its shaft and the wheel bored to fit on to an intermediate shaft. In heavy automobiles the rear-axle drive is usually a pair of worm gears, such as is shown in Fig. 54. This type of gear is specially suited to the comparatively high reduction ratios (between 6 to 1 and 10 to 1) required in such drives, and its quiet running qualities are specially valuable in trolley-buses because in that class of vehicle all other noises have been reduced to a negligible total.

Automobile Change Gears.—These are now so accurately cut that the noisiness associated with them is a thing of the past. This is due not only to the use of helical gear teeth but also to ground teeth, the use of shafts of stouter diameter which do not whip, and to improved methods of mounting.

The use of constant mesh gears instead of sliding dogs has also contributed to gearbox efficiency.

In the synchromesh gearbox the constant mesh gears are fixed on the primary shaft and countershaft as also are all the countershaft gears, but the second and third mainshaft gears are free to revolve on the mainshaft, and are engaged by means of dogs formed on the gears.

It must be remembered that in replacing worn gears it is advisable to renew them in pairs, as replacement of one of the pair invariably tends to create noise and cause accelerated wear of the new gear.

Mention must also be made of the overdrive. One type takes the form of two bevel pinions in the rear axle, meshing with crown-wheels of different diameter. Either of the bevel pinions can be locked to the driving shaft by means of a dog clutch, so that either a high or a low back-axle ratio is available. It will be seen that with this arrangement, not only are two top gears available but also a low and high ratio for each of the other gears.

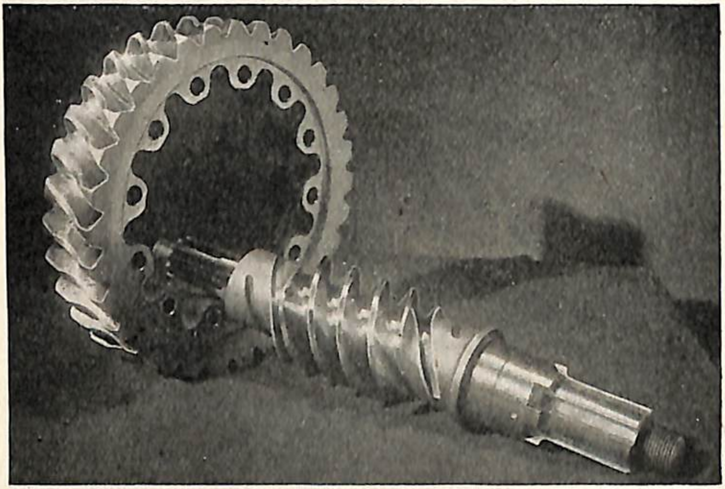


FIG. 54.—Worm gears for automobile rear-axle drive.

The same effect is obtained in the case of certain large passenger and commercial vehicles by providing an auxiliary two-speed box behind the gearbox. A further alternative is to provide an additional ratio providing an overgeared top in the gearbox itself.

CHAPTER VI

EPICYCLIC GEAR TRAINS

A GEAR train in which any gear rotates about a centre line which is not fixed is called an "epicyclic" train. In the example in Fig. 55 the gear B runs on a pin carried by arm D which rotates about the axis XX. Gear A is carried by another shaft whose centre line is also XX. Gears A and B mesh together, and B also meshes with the internal gear C whose centre line is also XX. If C is prevented from rotating, rotation of A will cause rotation of D with a definite velocity ratio. Similarly, if any one of the members A, B, C, and D is prevented from rotating, the motions of the other three are governed by fixed relations. In the following table, each of the letters A, B, C, and D refers to the number of teeth in the corresponding gear:

	Revolutions of			
	Gear A	Gear B	Gear C	Arm D
Case 1. D fixed	1	$-\frac{A}{B}$	$-\frac{A}{C}$	0
Case 2. C fixed	$1+\frac{A}{C}$	$\frac{A}{C}-\frac{A}{B}$	0	$\frac{A}{C}$
Case 3. B fixed	$1+\frac{A}{B}$	0	$\frac{A}{B}-\frac{A}{C}$	$\frac{A}{B}$
Case 4. A fixed	0	$\frac{A}{B}+1$	$\frac{A}{C}+1$	1

Case 3 is included for the sake of completeness, although it is unlikely to occur in practice. The condition is that B is prevented from rotating although permitted to move in a circle in accordance with the rotation of D.

To illustrate the use of the table, consider the assembly in which

$$A=20 \quad B=18 \quad C=56$$

and C is fixed. Then

$$\text{A makes } 1+\frac{20}{56} = 1{\cdot}357 \text{ revolutions while}$$

$$\text{B makes } \frac{20}{56}-\frac{20}{18} = -0{\cdot}754 \text{ revolutions and}$$

$$\text{D makes } \frac{20}{56} = 0{\cdot}357 \text{ revolutions}$$

If A is the driving gear, and D the driven member, then

$$\text{Velocity ratio} = \frac{1{\cdot}357}{0{\cdot}357}=3{\cdot}8.$$

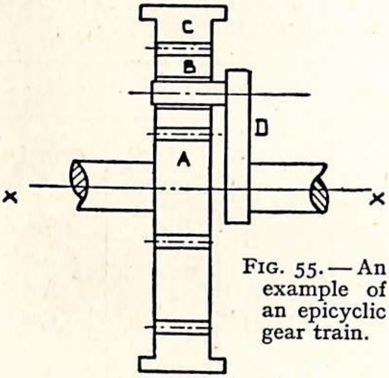


Fig. 55.—An example of an epicyclic gear train.

Gear A is called the "sun," and B is a "planet." In order to obtain greater load-capacity and to improve the balance of D, more than one planet may be used. Such an arrangement cannot be assembled, however, unless (A+C) is exactly divisible by the number of planets.

Compound epicyclic gear trains are often used for obtaining very high velocity ratios. The revolutions of the gears in the particular compound train, shown in Fig. 56, are given, for several important cases, in the table overleaf.

Case 2 is the one most frequently used in practice, with A the driving gear and E the driven gear. If B and C are nearly equal, the velocity ratio between A and E becomes very high.

The method of constructing tables, such as those given above, is simple and is applicable to any epicyclic gear assembly. In the first instance, the member carrying the

	A	B and C	D	E	F
Case 1. F fixed	1	$-\frac{A}{B}$	$-\frac{A}{D}$	$-\frac{A}{B}\frac{C}{E}$	0
Case 2. D fixed	$1+\frac{A}{D}$	$\frac{A}{D}-\frac{A}{B}$	0	$\frac{A}{D}-\frac{A}{B}\frac{C}{E}$	$\frac{A}{D}$
Case 3. A fixed	0	$\frac{A}{B}+1$	$\frac{A}{D}+1$	$\frac{A}{B}\frac{C}{E}+1$	1
Case 4. E fixed	$1+\frac{A}{B}\frac{C}{E}$	$\frac{A}{B}\left(\frac{C}{E}-1\right)$	$\frac{A}{B}\frac{C}{E}-\frac{A}{D}$	0	$\frac{A}{B}\frac{C}{E}$

Table showing the revolutions of the gears in the compound train illustrated in Fig. 74.

planetary spindle is assumed to be fixed. Any other member is imagined to have one revolution, and the corresponding revolutions of the remaining members are determined exactly as for an ordinary (not epicyclic) train. This is Case 1. Now if any particular member is to be considered at rest, the numbers of revolutions of all members are each reduced by the amount necessary to make that particular member's revolutions equal to zero. Thus, in the first table, Case 2 is derived from Case 1 by subtracting

$$-\frac{A}{D}\ \left(\textit{i.e.}\text{ adding }\frac{A}{D}\right)\text{ to each quantity.}$$

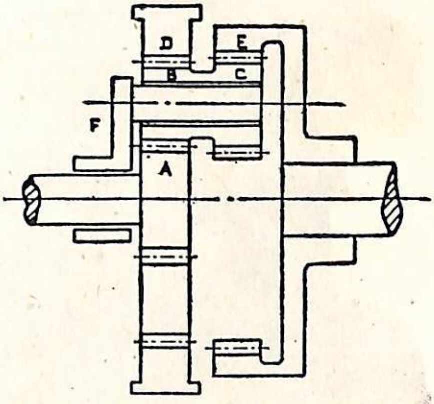


Fig. 56.—A compound epicyclic gear train.

Although the compound epicyclic gear is a means of obtaining a high velocity ratio in small space, its efficiency is low if the ratio is high, and on that account it is suitable only for small powers.

CHAPTER VII

METHODS OF MOUNTING

GEARS have to be manufactured to a high degree of accuracy for two main reasons:

(a) to permit of reasonably quiet running, and
(b) to permit of adequate distribution of load across the face-width.

The first of these requires that errors of tooth shape be kept within two or three ten thousandths of an inch, that pitch errors between adjacent teeth be less than about half a thousandth of an inch, and that the pitch error between any two teeth in the gear be less than one or two thousandths of an inch. A reasonable approximation to uniformity of distribution of load requires that the tooth

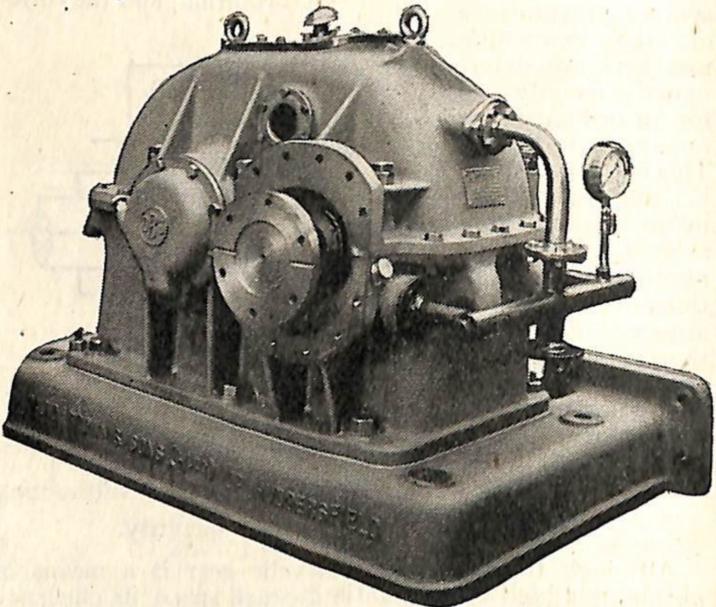


FIG. 57.—A self-contained single-reduction turbine gear unit (hobbed double-helical gears).

spiral shall be correct within about one thousandth of an inch across the width of the gear.

These degrees of accuracy can be achieved by appropriate care in manufacture, but they are quite unavailing unless the gears are mounted with equal accuracy. In some instances, particularly highly-stressed units such as automobile rear-axle assemblies, distortion of the mounting under the bearing-loads produced by the gears is sufficient to affect the accuracy of meshing to a serious extent, and the initial "setting-up" has to be adjusted so as to counteract the full-load distortion. Such adjustment means that spread of contract over the whole face-width can occur only at full load or some selected fraction of full load and emphasises the desirability of the utmost rigidity in the mounting.

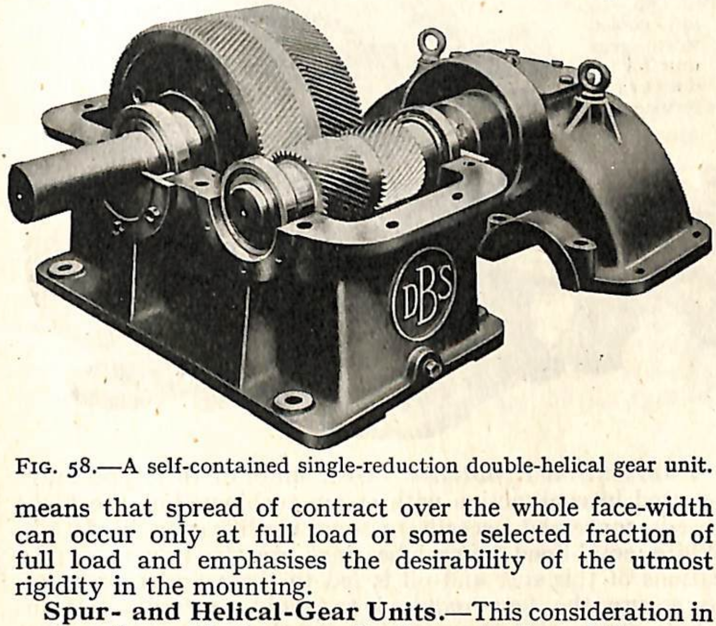
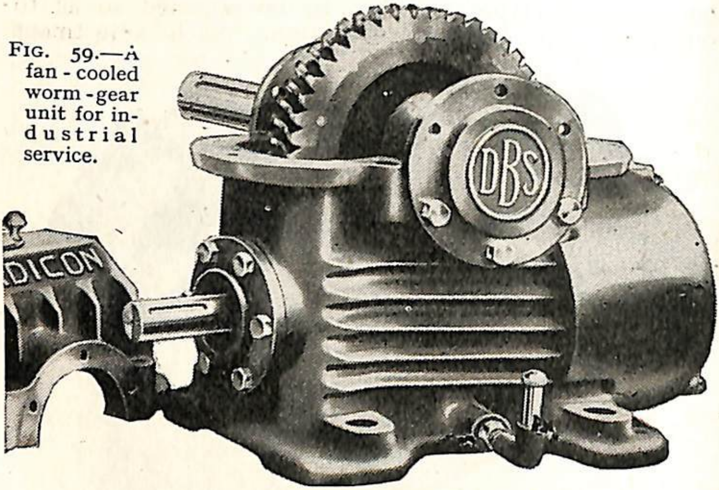


FIG. 58.—A self-contained single-reduction double-helical gear unit.

Spur- and Helical-Gear Units.—This consideration in conjunction with the necessity for protection of the gears from dust and dirt and for their lubrication has led to the use of self-contained gear units. These are standardised in type and size so that manufacturing costs are minimised. Bearing details and lubricating arrangements are settled once and for all, and the only variations between individual units lie in the gear ratio.

The parallel-shaft type of gear unit (Fig. 58) may contain spur, single-helical or double-helical gears and the bearings are usually of the ball or roller type. Unless the peripheral speed is higher than about 2500 feet per minute, lubrication of the gears is effected by allowing them to dip in a bath of oil contained in the lower part of the case. Oil is splashed on to the walls of the case, and troughs are provided to guide into the bearings the small amount of lubricant that they need.

FIG. 59.—A fan-cooled worm-gear unit for industrial service.



Lubrication Problems.—Gear units of this type may be used in conjunction with steam turbines, but the high speeds concerned necessitate some modification in detail. White-metal-lined journal bearings are the rule in applications of this sort and oil is fed to them under pressure to ensure the flow required to avoid excessive rise in temperature. Oil is sprayed on to the gears near to the point of mesh, oil bath lubrication being objectionable owing to the considerable power loss which occurs at very high peripheral speeds. In turbine reduction gears the peripheral speed may be as high as 15,000 feet per minute.

Bevel-Gear Units.—For connecting shafts whose centre-lines are perpendicular, either bevel-gear or worm-gear units may be used, the former having some preference if the gear ratio required is lower than about 4 to 1. An example is shown in Fig. 7, the horizontal shaft being driven by an internal-combustion engine and the vertical shaft transmitting power to a centrifugal pump situated some distance below the engine-room. Installations of a similar character are associated with deep bore-hole pumps, whose speeds are often three or four times as high as those of the engines which supply the power.

Worm-Gear Units.—Largely because of the wide range of ratios that it can conveniently afford the worm drive is extensively used in industrial service. It also gives high torque capacity in small space and is consequently an inexpensive form of gear transmission. Its mechanical efficiency is lower than that of a helical-gear unit, but this disadvantage is insufficient to outweigh the worm gear's compactness and low initial cost. A rough estimate of the overall efficiency of a worm-gear unit may be obtained by subtracting half the reduction ratio from 100; this rule apples for ratios between about 10 to 1 and 50 to 1.

It is very often found that the load capacity of a worm-gear unit with natural cooling is limited by temperature-rise rather than by endurance of the teeth. In order to raise this limitation it is now usual to provide artificial cooling by mounting a fan on the worm shaft and arranging to direct its draught between fins cast on the gearcasing. Fig. 59 illustrates a typical worm-gear unit of this type.

CHAPTER VIII

MEASURING GEARS

A GEAR dimension that is invariably checked either directly or otherwise is the tooth thickness. Given an accurate machine and cutter, the desired tooth thickness may be secured by sinking the cutter to the appropriate depth in the blank, and whilst in practice this is often done (with the aid of measuring devices built into the machine and indicating the movement of cutter-mounting relatively to the blank), the final check is usually made by means of a gear-tooth caliper (Fig. 60). This has a vernier adjustment, setting the edge of a blade at the required height above two gauging points. The distance between these points is measured by a second vernier. The caliper thus measures the thickness of a tooth in a plane at any required distance from the tip of the tooth.

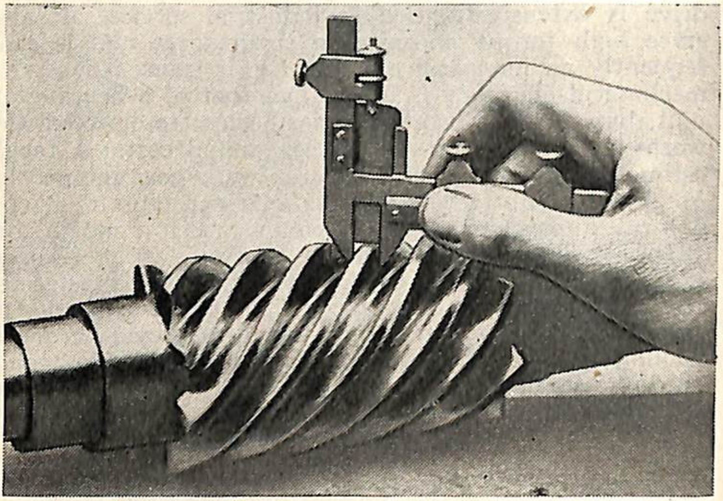


FIG. 60.—A vernier caliper for measuring the thickness of gear teeth.

The vernier graduations correspond to steps of ·001 in., but owing to the difficulty of reading such fine graduations it is not usually possible to guarantee a setting to less than

·002 in. Even this order of accuracy more than meets normal requirements, any error occurring on this account being easily absorbed in the backlash allowance which is rarely less than about 0·005 in.

The gear-tooth caliper may be applied to any type of gear tooth and usually, though not essentially, it is set to measure the tooth thickness on the pitch cylinder of generation.

Micrometer Measurement of Tooth Thickness.—In certain circumstances the tooth thickness of involute spur gears may be determined from micrometer measurement over the teeth. In Fig. 61 are shown a number of tangents to the base circle of a spur gear. From the construction of the involute it follows that each of the tangents shown intersects the two "opposed" involutes at right angles. Also the distance between the points of intersection of a tangent with the two involutes is equal

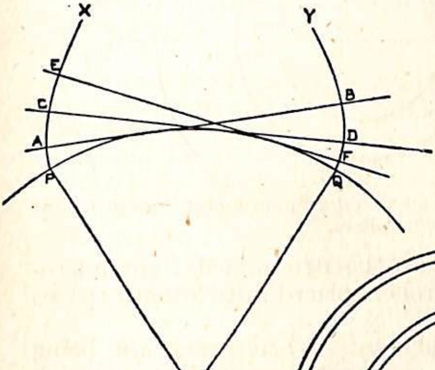


Fig. 61.—Opposed involutes and tangents to a base circle.

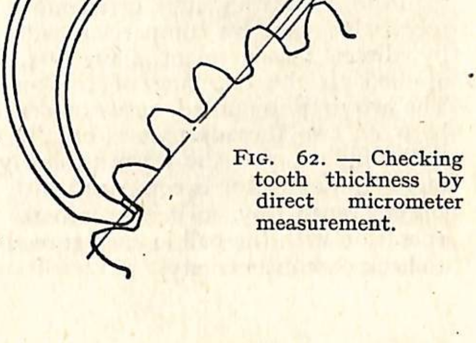


Fig. 62. — Checking tooth thickness by direct micrometer measurement.

to the distance PQ (measured round the base circle) between the origins of the two involutes. Hence the distances AB, CD, EF are all equal and consequently a micrometer measurement over any two "high points" such as A and B on the opposed involutes X and Y gives the same dimension (see Fig. 62). This can be calculated from the tooth thickness and the number of teeth between X and Y. Obviously this method can be used only where the dimensions of the gear are such that a tangent to the base circle can be drawn to intersect two involute profiles in points to which a micrometer can be applied. Alternatively use may be made of a vernier caliper, but this is less accurate than a micrometer.

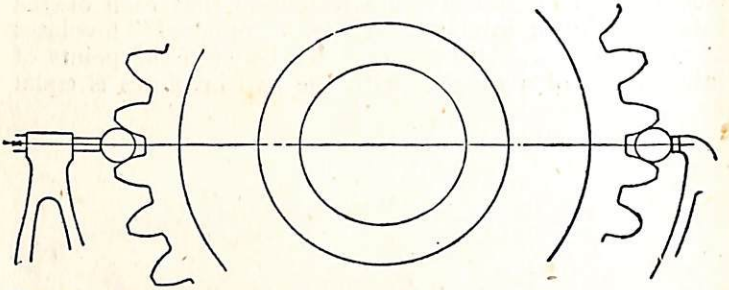


FIG. 63.—Checking tooth thickness by micrometer measurement over rollers.

Tooth thickness may also be determined from micrometer measurement over rollers placed in two tooth spaces (see Fig. 63).

Checking by Comparison.—When gears are being made in quantities, it is often more convenient to check tooth thickness by comparison with a master gear than by direct measurement. Fig. 64 shows this principle applied to the checking of thickness of worm threads. The worm is mounted between centres, and a ball held between two threads presses on the flat gauging anvil of a dial indicator as the worm is slowly rotated. The reading of the indicator is compared with that given in similar circumstances by a master worm. Repetition of the operation with the ball in each thread-space in turn affords a check on concentricity.

For checking the thickness of worm-wheel teeth a ball micrometer (Fig. 64) is often used; here again the reading obtained is compared with that derived from a master gear.

Tooth Shape.—The simplest method of checking the tooth shape of spur, helical or straight bevel gears is to lay against a profile a correctly formed plate gauge. Such a gauge is made by filing until its optical projection (at 25 to 50 magnifications) matches an accurate large-scale drawing.

A comparable direct check on tooth form is afforded by using a gear-tooth caliper to measure tooth thickness on various planes between tip and root and comparing the figures with those obtained by calculation.

These methods are, however, insufficiently accurate for modern precision gears, and instruments have therefore been developed by which the departure of the profile from the desired form is directly indicated. An example is the instrument shown in Fig. 66. Here the gear under inspection is mounted on the same spindle as a disc of diameter equal to that of the base circle of the gear. The disc is pressed into contact with a straight-edge C by

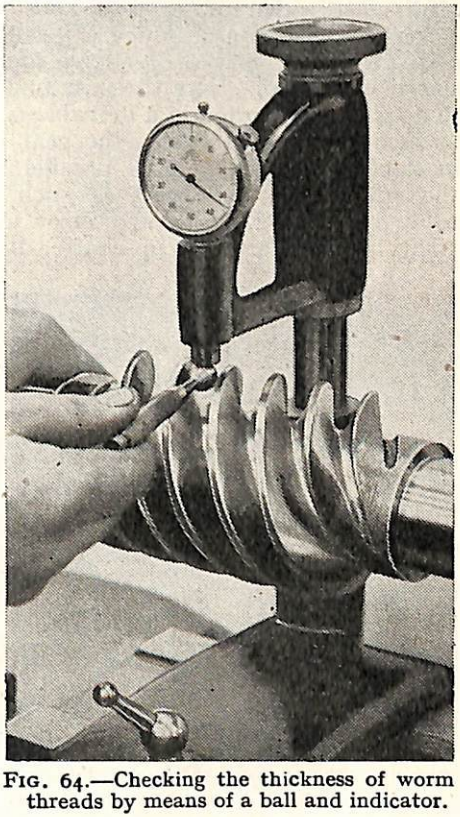


FIG. 64.—Checking the thickness of worm threads by means of a ball and indicator.

means of screws acting through the rods AA and the bar B. Endwise movement of the bar causes the gear to roll along the straight-edge and (looking parallel to the axis of the gear) the point of intersection of a true involute profile with the straight-edge will not change its position.

The checking of the profile is effected by a stylus placed so as to touch it on the line of the straight-edge and operating the plunger of a dial indicator reading in ten-thousandths of an inch. Departure of the tooth profile from the true involute form is shown directly by the reading of the dial indicator.

A rapid check on accuracy of profile and concentricity of teeth and bore is afforded by the type of gear-tester shown in Fig. 67. Here the gear under test is slipped on to a spindle supported in a movable carriage spring-loaded in such a way as to press the gear into contact with a master gear mounted on a parallel spindle. The fit of the spindles in the gears is such that they can be rotated by hand, when any error in profile or concentricity causes variation in centre distance between the

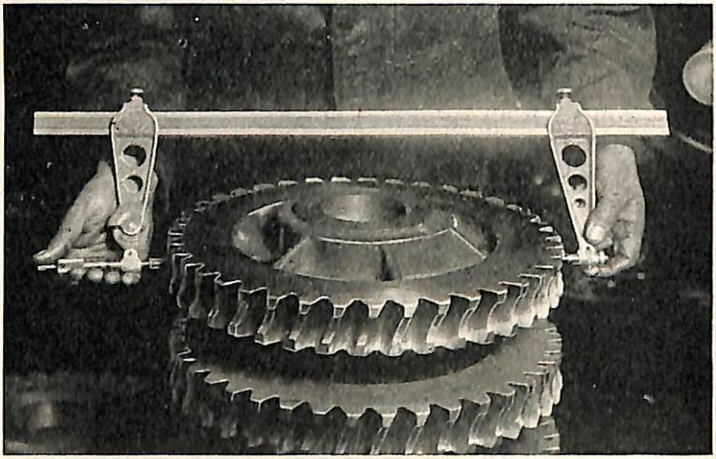


FIG. 65.—Checking the thickness of worm-wheel teeth by the use of a ball micrometer.

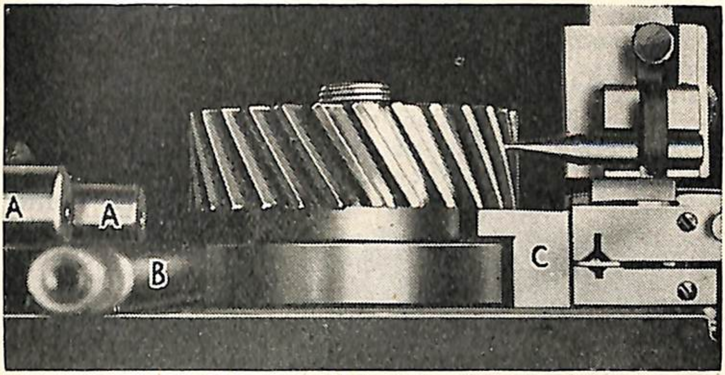


FIG. 66.—An involute-testing instrument.

spindles. This variation is shown on a dial indicator, and a permament record may be taken by a pen which traces a line on a disc rotating in unison with the master gear.

A perfect combination of gears results in a circular track; any variation in centre distance as the gears revolve is reproduced with suitable magnification as a radial departure from the ideal circular form.

The instrument shown in Fig. 67 is designed to accom-

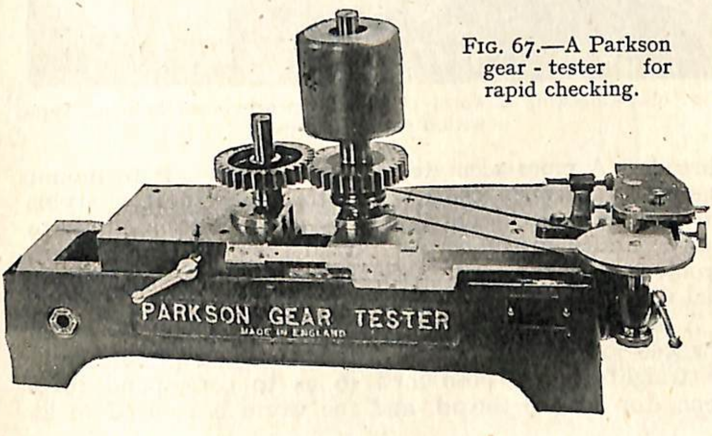


FIG. 67.—A Parkson gear - tester for rapid checking.

modate spur or helical gears. The same general principle applies in other machines arranged to take bevel gears and worm gears.

Profile Checking of Worm Gears.—Some worm-thread profiles show straight sections in planes passing through the centre-line of the worm. In such cases, checking of shape may be effected by a gauge fixed to a V-block placed in contact with the tips of the worm threads. A more accurate method, however, is to mount the worm between centres and to apply to it a stylus pivoted on a block constrained by an adjustable guide to move parallel to the correct thread profile. Any error in profile is detected by the stylus and is made visible on a dial indicator.

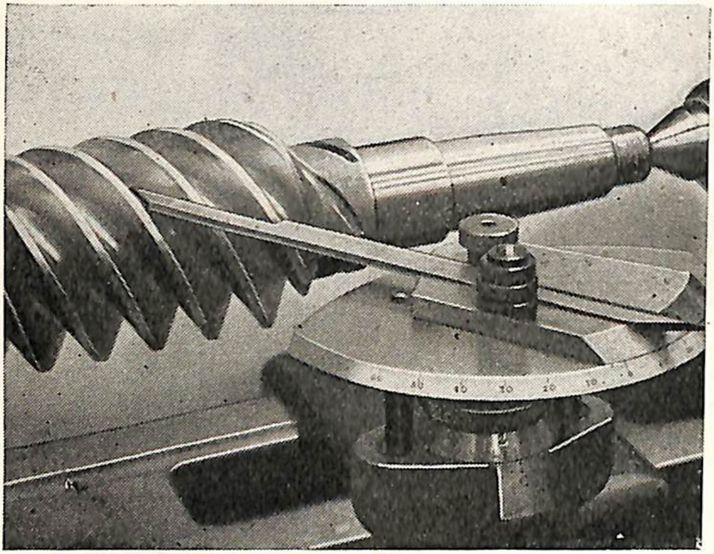


FIG. 68.—Checking a worm thread of the involute helicoid type with a straight-edge.

A worm thread of the involute helicoid type may be checked for shape in the manner shown in Fig. 68. Here a straight-edge is positioned so as to correspond to a generator of the thread, and the worm is rotated so as to bring a thread into contact with the straight-edge. Examination of the line of contact with a light placed on the far side of the straight-edge reveals profile errors which, if large enough, may be estimated by use of feeler gauges.

The form of worm-wheel teeth is so complex that the only practicable way of checking it is to mount the wheel in correct relation to a master worm whose threads have been lightly smeared with "blue marking" and after rotation of the gears to observe the distribution of marking on the wheel-teeth.

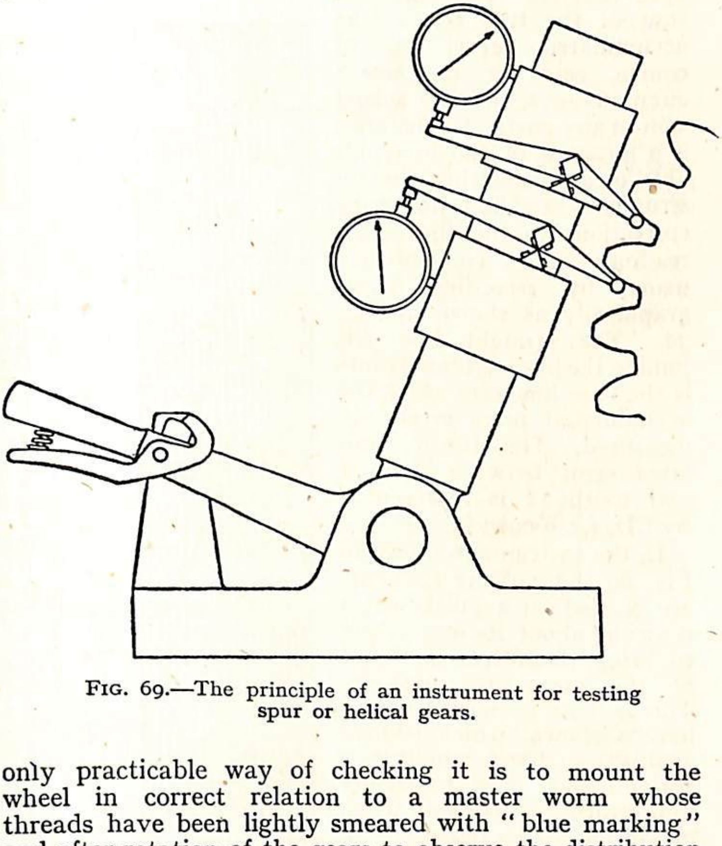


FIG. 69.—The principle of an instrument for testing spur or helical gears.

Pitch-Testing.—The principle involved in the pitch-testing instrument shown in Fig. 69 is to bring into contact with two gear teeth a pair of styluses mounted in such a way as to show on dial indicators the amount by which the distance between the gauging points differs from the original setting. These errors are recorded for consecutive pairs of teeth until the whole circumference of the gear has been covered. The algebraic sum of the true errors (the accumulated error) is, of course, zero for the whole circumference, and the actual sum in any particular instance is a measure of the unavoidable errors in setting the instrument. The necessary corrections to the individual readings are conveniently made by recording them graphically as shown in Fig. 70. The straight line AB joining the first and last points is the base line from which the accumulated pitch errors are measured. Thus the accumulated error between tooth 1 and tooth 12 is represented by CD, *i.e.* 0·0003 in.

In the instrument shown in Fig. 69 the gauging elements are carried on a pillar which is swung about its anchor-pin to bring them into and out of the measuring position. This is done by means of the handle shown, which is held against a fixed pin by a spring-loaded pawl to ensure uniformity of location when measuring. This movement of the gauging elements is necessary

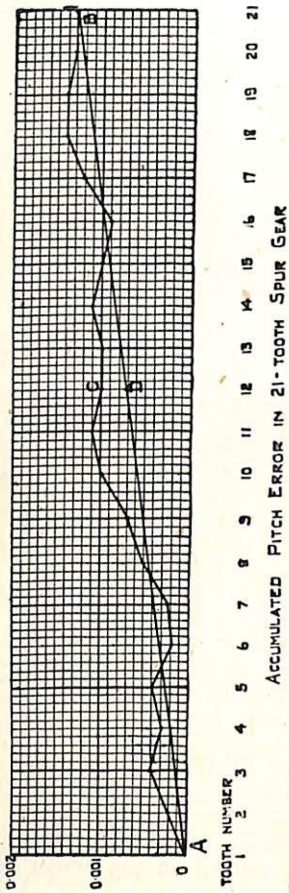


FIG. 70.—The corrections to the individual readings given by the instrument illustrated in Fig. 67 are made by recording them graphically as shown here.

to permit of rotation of the gear for successive readings.

In order to deal with large gears which could not conveniently be rotated about a fixed axis for measuring purposes, the pitch-testing instrument shown in Fig. 71 may be employed. The framework is held in contact with the tips of the teeth of the gear to ensure its correct location in relation to the centre-line of the gear. Circumferential location relative to a tooth is by means of a ball attached rigidly to the frame, whilst a dial indicator measures the adjacent error between any two teeth that are not too widely separated.

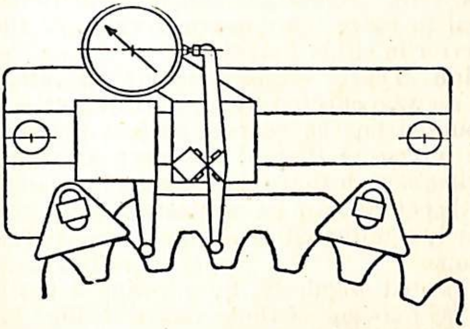


FIG. 71.—An instrument for measuring adjacent pitch errors in large gears.

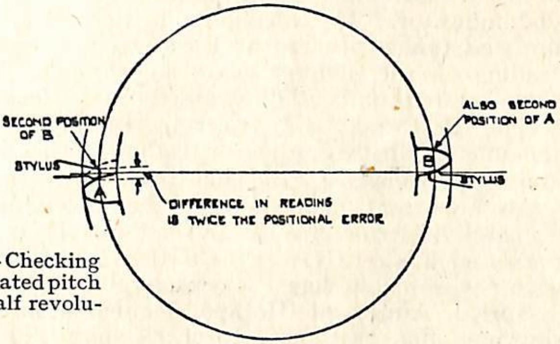

SECOND POSITION OF B
STYLUS
A
ALSO SECOND POSITION OF A
B
STYLUS
DIFFERENCE IN READING IS TWICE THE POSITIONAL ERROR

FIG. 72.—Checking the accumulated pitch error in a half revolution.

A check on accumulated error in gears which are small enough to be rotated on centres is shown in principle in

Fig. 72. The instruments are set to measure the accumulated error between two teeth nominally separated by 180 deg. By rotating the gear through an angle which causes the gauged teeth to interchange their positions, the instruments may be made to measure the errors in the two different half circles. The difference between these readings is equal to the difference between the true errors, whose sum must be zero. Hence the difference between the readings is equal to twice the numerical value of the accumulated pitch error in either half circle.

Angular Division Tester.—Measurement of pitch errors in gear teeth may be effected by an optical method. A theodolite is mounted on the gear in such a position that its centre-line intersects that of the gear at right-angles. The mounting is such that relative angular movement of gear and theodolite can be accurately measured by means of a finely graduated dividing plate and a "built-in" microscope.

The gear is first located angularly, by allowing a tooth to bear on a stylus, such as one of those shown in Fig. 69, accurate positioning being ensured by use of a sensitive dial indicator. The theodolite is sighted on to an illuminated target situated at least ten feet away, and the reading on the dividing plate is recorded. The gear is then rotated until a different tooth is located by the stylus, the theodolite is redirected on to the target, and the difference between the new reading on the dividing plate and the previous one is the angle between the selected teeth.

An important point is that this procedure measures accumulated error directly and not merely as the sum of measured adjacent errors. On this account the gain in accuracy of measurement is considerable.

Spiral Angle of Helical Gears.—The instrument shown in Fig. 73 is used to check spiral angle and concentricity of helical gears. It employs a gauging member in the form of a tooth of the basic rack corresponding to the gear teeth. This member is mounted at one end of a spindle which carries on its other end an arm having two projecting pins. These permit a micrometer or slip-gauges to be used to effect accurate angular setting of the gauging member, and when this has been done a dial indicator is set with its operating plunger in contact with one pin.

The spindle assembly is then moved bodily towards the gear until the gauging member has penetrated as far as possible into a tooth space. Any error in spiral angle acts on the gauging member to rotate the spindle, the amount of such movement being shown on the dial indicator.

The distance between the spindle mounting and the axis of the gear when the gauging member is fully engaged is measured with a second dial indicator, and variation of this reading from one tooth space to another is a measure of eccentricity.

Checking by Running Together.—The most troublesome effect of errors in high-speed gears is that of noisy running. Consequently, if it is convenient to mount a pair of gears in bearings and to run them at full speed, this is the most satisfactory method of checking them. If they pass such a running-test it may be concluded that any errors they do possess are too small to be of any practical importance

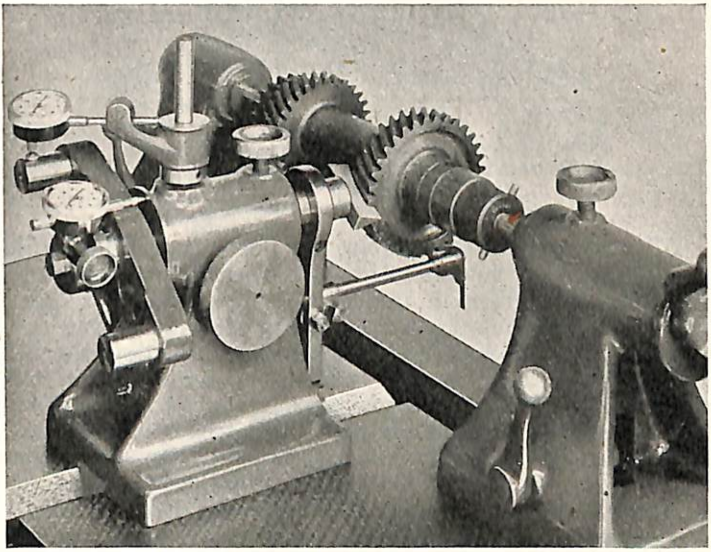


FIG. 73.—An instrument for checking the spiral angle and concentricity of helical gears.

Gear-Tooth Comparators. — Until comparatively recently the only means of measuring gear teeth was by means of the gear tooth-caliper, an instrument necessitating two settings for each pitch measured, and also a different setting for any variation in the number of teeth for a given pitch. That is to say, if a gear of 20 teeth, six d.p., is being measured, a different setting is required for a gear of 30 teeth of the same pitch. This complication arises out of the fact that the gear-tooth caliper measures the thickness at the pitch line, so that for gears of a given pitch, but of varying numbers of teeth, a calculation has to be made of the chordal thickness and also the chordal height.

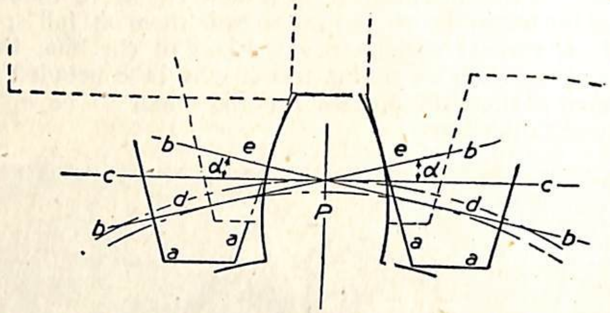


FIG. 74.—A diagram explaining the principle of the Sykes gear-tooth comparator or caliper.

All workmen, unfortunately, are not able to make this calculation, and they are rather apt to set the instrument to measurements given in tables, which, of course, require correction according to the number of teeth. Another disadvantage of this instrument is the fact that the measurement of the tooth thickness is taken on the corners of the two jaws, so that the wear is concentrated at these points and leads to a rapid deterioration of the instrument.

Two instruments which have largely overcome these difficulties are the Sykes' comparator and the Sykes' caliper for measuring gear teeth. The comparator and its method of application is shown in the accompanying illustrations.

It will be observed that, in fine, the instrument consists of a substantial frame or beam carrying one fixed and one adjustable jaw, in addition to a specially designed dial test indicator. The movable jaw is provided with means for fine adjustment, and between the jaws the plunger of the dial indicator projects. Fig. 75 shows the instrument set to a master gauge block. Fig. 78 shows the instrument complete in case, with a set of master gauge blocks.

The Working Principle.—The principle on which the instrument is based will be obvious to those possessing a knowledge of the property of the involute. Each gauge block represents an involute rack tooth of a particular pitch and pressure angle, and the inclined faces of the jaws are made to correspond. It is well known that any gear of a given pitch, irrespective of the number of teeth, will accurately gear with a rack of the same pitch and pressure angle. Therefore, when the comparator is set for a rack tooth (represented by the gauge block), the setting is correct for all teeth of the same thickness, pressure angle and addendum. It will readily be apparent that any tooth

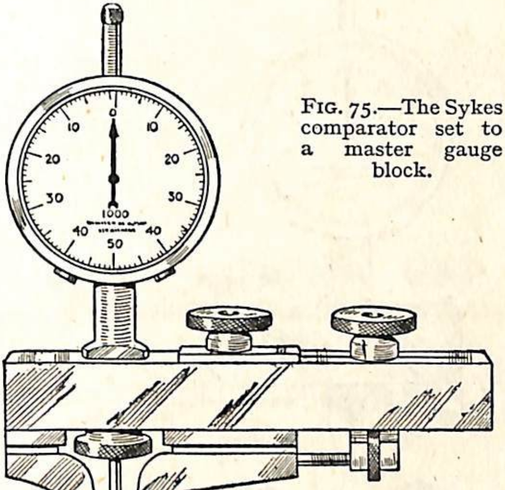


Fig. 75.—The Sykes comparator set to a master gauge block.

thicker or thinner than that setting will cause the dial indicator to show plus or minus, owing to the teeth entering to a lesser or greater distance between the jaws.

Using the Comparator.—For convenience the dial of the indicator can be rotated so as to alter the position of the zero mark. When it is only desired to test teeth for uniformity, gauge blocks may be dispensed with, as a master gear may be used to set the comparator. The jaws of the instrument and also the gauge blocks are glass hard and are ground and lapped to a minute degree of error.

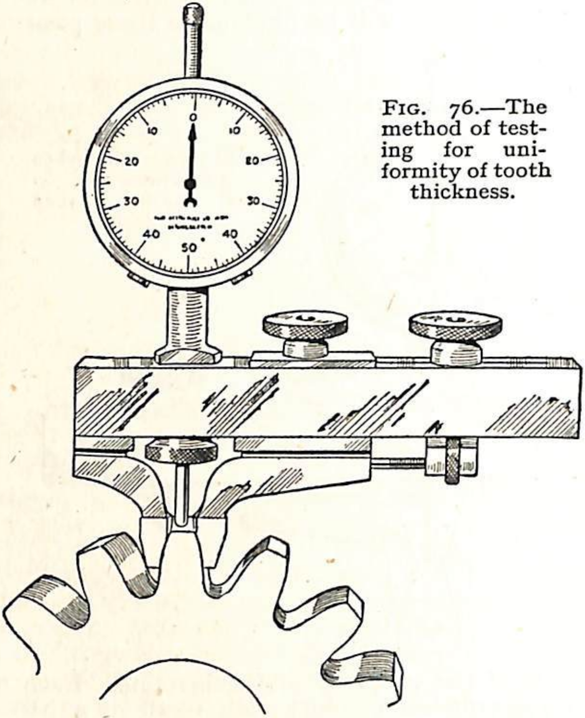


Fig. 76.—The method of testing for uniformity of tooth thickness.

The standard gauge blocks are supplied at 14½ degrees pressure angle, but of course, blocks of any pressure angle can be supplied.

One division on the dial represents ·0005 in. for teeth of 14½ degree pressure angle. The instrument has a capacity for 12 d.p. to 1 d.p., or from ¼ in. to 3 in. circular pitch.

It must not be assumed that a gauge block is necessary for every pitch to be measured. Seven blocks will measure all pitches; six blocks will measure all pitches between 12 d.p. and 1½ d.p. and four blocks will measure every pitch between 12 d.p. and 3 d.p.

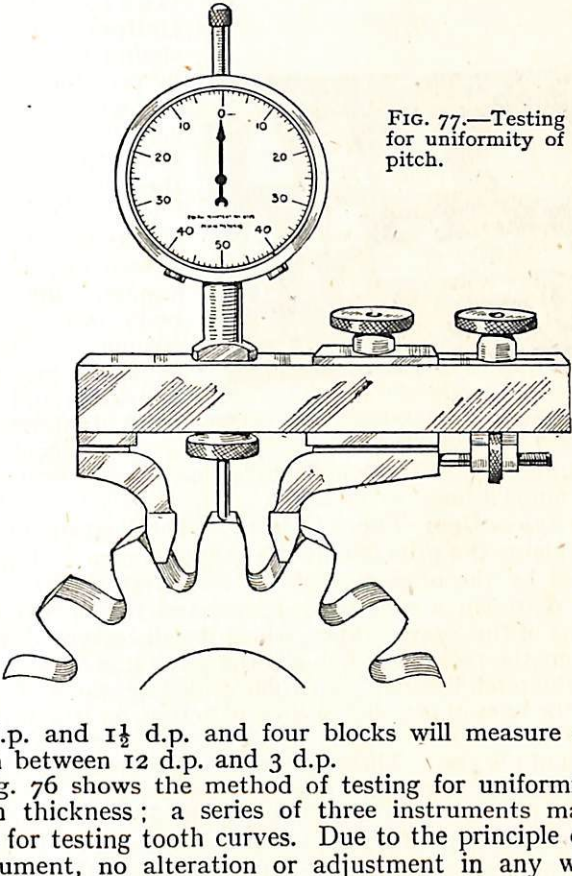


FIG. 77.—Testing for uniformity of pitch.

Fig. 76 shows the method of testing for uniformity of tooth thickness; a series of three instruments may be used for testing tooth curves. Due to the principle of the instrument, no alteration or adjustment in any way is necessary in cases where the addendum of the pinion is increased or decreased, provided that the addendum of the wheel into which the pinion gears is similarly decreased or increased. In either case the jaws automatically bear on the theoretical contact points of the tooth curves.

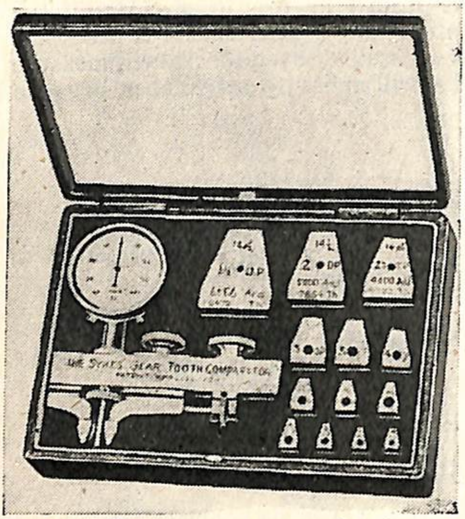


FIG. 78.—The comparator complete in case, with a set of master gauge blocks.

Zeiss Optical Gear - Tooth Caliper.—This instrument is similar in principle to the vernier gear-tooth caliper but differs from it in that the " height " and " thickness " scales are on glass and can be seen only through a magnifying glass built into the instrument. They lie at right angles to each other and their point of intersection is at the centre of the field of view. The longitudinal line of each scale acts as the fiducial line for the other.

The Sykes Gear-Tooth Caliper.—The diagram, Fig. 74, also explains the principle of the caliper shown in Fig. 79, designed by the originator of the comparator. The lines *aa*, *aa* represent a rack-tooth space, and the dotted lines the jaws of the Sykes caliper, which it will be seen directly represent the rack. The line *c* is the pitch line of the rack, and *d* the pitch line of a particular pinion or wheel. *bb* and *bb* are the lines of pressure or lines of action, on which tooth contact must always occur, whatever the size or number of teeth of the gear. These lines are always the same for any given pressure angle, and it is well known that in the involute system the rack tooth has straight sides, and the first law of gear-tooth contact is the common normal to the tooth curves must pass through the pitch point. In the diagram the pitch point is *p*, and the lines *b* pass through it and are normal to the rack-tooth profile. It will, therefore, be apparent that contact must always take place on the lines *b*, and that the position of the point of contact *e* depends only on the thickness of the tooth. In gears of any particular pitch the profile of the tooth is different for each number of teeth, but the point of contact, with the centre-line of the tooth and tooth space coincident, never varies.

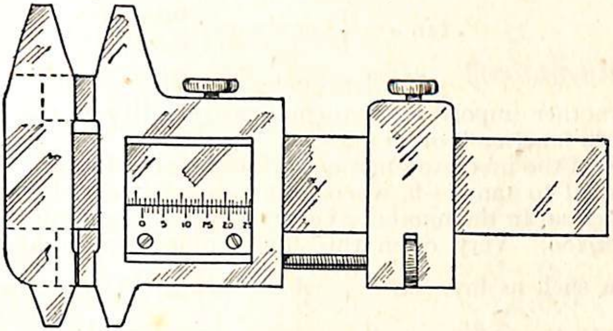


FIG. 79.—The Sykes gear-tooth caliper.

Base Circle.—In measuring helical or spur-gear teeth an important dimension is the base diameter of the gear. This is denoted by d_o and its value is given by

$$d_o = \frac{tp_t}{\pi} \cos \psi_t \text{ or } \frac{tp_n}{\pi} \frac{1}{\sqrt{(\cos^2 \sigma + \tan^2 \psi_n)}}$$

where t = number of teeth in gear.

p_t = transverse pitch of teeth (measured in plane perpendicular to axis).

p_n = normal pitch of teeth (measured in plane perpendicular to tooth spiral).

ψ_t = transverse pressure angle (measured in plane perpendicular to axis).

ψ_n = normal pressure angle (measured in plane perpendicular to tooth spiral).

σ = helix angle at pitch cylinder.

σ_o = helix angle at base cylinder.

L = Lead of helix.

The base helix angle σ is determined from the given quantities by the relation

$$\sin \sigma_0 = \sin \sigma \cos \psi_n$$

and

$$\tan \sigma = \frac{tp_t}{L} \text{ or } \sin \sigma = \frac{tp_n}{L}$$

also

$$\tan \psi_t = \tan \psi_n \sec \sigma.$$

Another important mathematical quantity is the "involute function" of an angle.

Thus the involute function of θ is denoted by inv. θ and is equal to $\tan \theta - \theta$, where θ is measured in radians, *i.e.* it is equal to the number of degrees in the angle multiplied by $\pi/180$. Very often this function is introduced in a form such as inv. $\sec^{-1} \frac{d}{d_0}$ which means "Determine the angle whose secant is $\frac{d}{d_0}$ (or whose cosine is $\frac{d_0}{d}$) and then find the involute function of that angle by subtracting its value in radians from the value of its tangent."

To determine the tooth thickness of involute spur or helical gears from micrometer measurement over the teeth (see Fig. 62):

Let t=number of teeth in gear.
T=number of teeth contained between micrometer anvils.
M=micrometer measurement over teeth.
W=tooth thickness measured perpendicular to tooth helix on cylinder of diameter d_1.
σ_1=helix angle at diameter d_1.
L=lead.
d_0=base diameter.
σ_0=helix angle at base cylinder.

Then

$$\tan \sigma_1 = \frac{\pi d_1}{L}$$

and

$$W = \left\{ \frac{M}{d_0 \cos \sigma_0} - \text{inv. } \sec^{-1} \frac{d}{d_0} - \frac{\pi}{t} (T-1) \right\} d_1 \cos \sigma_1$$

This gives the normal tooth thickness on any desired cylinder.

Determination of Tooth Thickness of Involute Spur Gears from Micrometer Measurement over Rollers (see Fig. 63):

Let d = diameter of circle on which tooth thickness is measured.

d_o = diameter of base circle.

Z = measurement over two equal rollers in opposite tooth spaces.

D = diameter of each roller.

W = circumferential thickness of tooth on circle of diameter d.

t = number of teeth in gear (must be even).

Then

$$W=d\left(\frac{\pi}{t}+\text{inv. }\sec^{-1}\frac{Z-D}{d_o}-\text{inv. }\sec^{-1}\frac{d}{d_o}-\frac{D}{d_o}\right)$$

Assuming that the profile of the teeth is accurately involute, this formula applies with any roller diameter that causes the roller to make contact with the teeth at points on the involute part of the profile.

If W is the *required* tooth thickness at diameter d, the calculations can be simplified by making D equal to $d_o\frac{\pi}{t}-W\frac{d^{o}}{d}$.

If the teeth have the required thickness, the dimension Z over two rollers of this size is D+d. If t is an odd number, and the rollers are placed in spaces as nearly as possible opposite to each other, the dimension over them is $D=d\left(1-\sin^2\frac{45^\circ}{t}\right)$.

Measurement of Tooth Thickness by Vernier Caliper (Fig. 60).—If tooth and space are to be of equal width at the pitch cylinder, the vernier caliper will fit the teeth with the settings specified below—

$$\text{Height Setting}=\text{Addendum}-0{\cdot}196\,\frac{p_n}{t}$$

$$\text{Thickness Setting}=\frac{tp_n}{\pi}\sin\left(\frac{90}{t}\right)^\circ$$

where p_n = normal pitch
and t = number of teeth in gear.

Alternative Formulæ.—As an alternative to the measurement formulæ given, the following formulæ are offered as being simpler to apply. The fundamental difference is that they apply to measurement of tooth thickness at points which do not lie on the pitch cylinder. The formulæ do not involve any mathematical approximation but are quite accurate.

To determine the tooth thickness of involute spur or helical gears from micrometer measurement over teeth by the base tangent method (Fig. 62):

Let p_n=normal pitch.
ψ_n=normal pressure angle.
ψ_t=transverse pressure angle.
σ=spiral angle.
t=number of teeth.
L=lead.
w=normal thickness of tooth at pitch cylinder.
n=number of teeth between micrometer anvils.
M=micrometer measurement over teeth.

Then

$$\sin\sigma = t\,p_n/L$$

$$\tan\psi_t = \tan\psi_n \sec\sigma$$

and $$M = \cos\psi_n \left\{ w + p_n \left[(n-1) + \frac{t}{\pi}\,\text{inv.}\,\psi_t \right] \right\}$$

Here inv. $\psi_t = \tan\psi_t - \psi_t$, the angle ψ_t being measured in radians, *i.e.* $(\pi/180)$ times the angle measured in degrees and decimals of a degree.

To determine the tooth thickness of involute spur or helical gears by micrometer measurement over rollers (Fig. 63):

The symbols are as above, except that w represents the *required* normal thickness of tooth at the pitch cylinder and then

$$\text{Roller diameter} = (p_n - w)\cos\psi_n$$

and the micrometer measurement over the rollers, when the normal tooth thickness is actually w, is given by

$$Z = \frac{t\,p_n}{\pi}\sec\sigma + (p_n - w)\cos\psi_n$$

This applies only to gears with even numbers of teeth.

To determine the tooth thickness of involute spur gears of odd numbers of teeth by micrometer measurement over rollers:

The symbols are as above, and micrometer measurement

$$Z=\frac{t\,p_n}{\pi}\left(1-2\sin^2\frac{45^\circ}{t}\right)+(p_n-w)\cos\psi_n.$$

This applies to straight tooth gears (so that p_n=circular pitch and ψ_n=transverse pressure angle) and to the condition in which one roller is in a tooth space as nearly as possible opposite to that occupied by the other roller. The number of teeth must be *odd*, and w represents the *required* tooth thickness at the pitch cylinder.

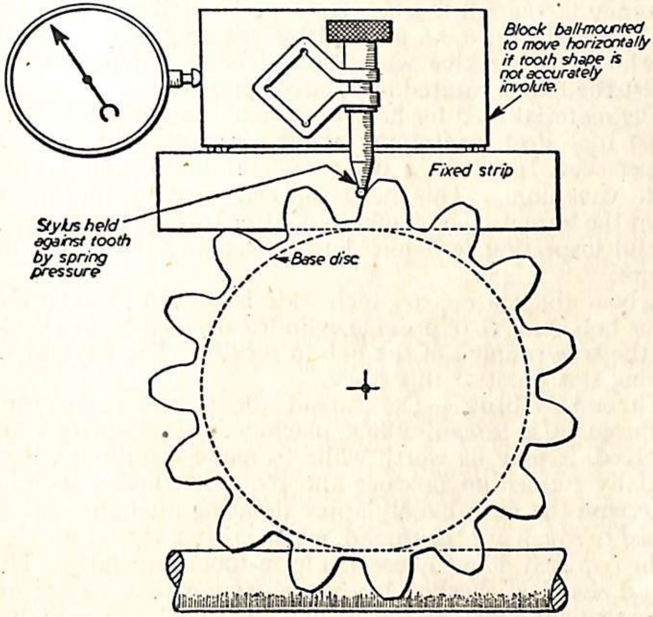

Block ball-mounted to move horizontally if tooth shape is not accurately involute.
Fixed strip
Stylus held against tooth by spring pressure
Base disc

FIG. 79A.—Diagram showing the principle of the involute gear-testing instrument. See also Fig. 66.

Micrometer measurement over two rollers cannot be applied to helical gears with odd numbers of teeth.

CHAPTER IX

HOBS, END-MILLS AND GENERATING CUTTERS

THE hobbing process is the one that produces the most accurate cut gears, and it does demand a high standard of accuracy in the hob itself. To meet modern requirements the hob teeth must be finished by grinding on a machine in which the abrasive wheel head has a relieving motion whilst the hob is rotated and moved endwise.

The material used for hobs of normal dimensions is high-speed tool steel, containing about 0·75 per cent. carbon, 18 per cent. tungsten, 4 per cent. chromium, and 1·3 per cent. vanadium. This needs special care in forging to avoid the formation of cracks and, after turning and boring, careful inspection is required to detect any defect of this nature.

A boss about a quarter-inch wide is turned at each end of the hob in order to provide cylindrical surfaces for checking the true running of the hob in service. The keyway or driving slot is cut at this stage.

Thread-Milling.—The thread (or threads) are then produced in a thread-milling machine. If quantities are involved, it may be worth while to make a milling cutter specially formed to produce the required thread section. Otherwise the most closely approximating available cutter is used to rough out the thread, which is afterwards modified to the required shape by use of a form-tool in the lathe. The thread shape is checked by comparison with a gauge or, if it is straight-sided, by the use of a sensitive tracer and dial indicator traversed in a straight line along a slide at the appropriate angle.

Milling Flutes.—The flutes are then produced on a universal milling machine using a cutter of 30 deg. V-section, one side of which is perpendicular to the axis of the cutter. The vertical line through the plane containing these perpendicular edges is set to intersect the centre-line of the hob flank so that the cutting face of each flute contains radial straight lines. The radius at the tip of the cutter is equal to about one-eighth of the normal pitch of the hob.

The angular setting of the table of the milling machine is the spiral angle of the flute at the pitch cylinder of the hob. The change-gears between dividing head and horizontal feed screw are selected to produce the lead of the flutes.

The hob is indexed from one flute to the next by use of the dividing head. The cutter is finally sunk into the blank so that the root of the flute is at a distance from the outer surface of the hob blank equal to

Depth of hob tooth + rise of relieving cam + tip radius of cutter.

Relieving.—The relieving of the teeth is carried out on a relieving lathe; the tool is formed to match the thread form of the hob, a witness about 0·020 in. wide being left on each tooth.

The hob is then hardened in accordance with the steelmaker's instructions for the particular brand of steel used, double-quenching and tempering frequently being specified.

Grinding.—The next operation is to grind the bore, end faces and cylindrical registering surfaces. These surfaces are required to run truly in relation to each other to an accuracy of 0·0001 in., and the diameter of the bore must not exceed the nominal size by more than 0·0002 in.

The cutting faces of the flutes are next ground on a hob-sharpening machine set up for the lead of flute specified for the hob. Indexing of the flutes is required to be accurate within 0·0005 in. measured at the pitch cylinder.

The relieved surfaces of the teeth are then ground on a hob-profile grinding machine. After the tip (which is usually straight) has been treated, the abrasive wheel is trimmed by successive trial to give the required hob tooth form. If this is of simple straight-sided type it may be checked by use of a fixture arranged to run a tracer-point along the required path. Otherwise an optical projector is used to compare a magnified shadow of the tooth with an accurate drawing of the required normal section on the same enlarged scale.

After one set of flanks has been satisfactorily shaped, the others are similarly treated. Further grinding is then done to bring the teeth to the required thickness.

The error of axial position of any tooth relative to its neighbours should not exceed 0·0002 in. and the error of relative position between any two teeth should not exceed 0·0002 in. plus 0·0001 in. for every inch of axial distance between them.

Sharpening.—The hob is sharpened when necessary by grinding the radial faces of the flutes on a hob-sharpening machine, accurately set up to give correct lead and indexing. This operation must be carried out with care if the hob is to reproduce the accuracy with which it was manufactured.

Formulæ used in Hob Design and Manufacture

Let p_n = normal pitch of tooth to be produced.
t = number of threads in hob.
d = pitch diameter of hob.
j = outside diameter of hob.
λ = lead angle of hob thread.
σ = spiral angle of flutes.
F = number of flutes.
L = lead of threads.
l = lead of flutes.

For hob made with solid shank
Minimum desirable value of $d = 4p_n$.

For hob bored to mount on arbor
Minimum desirable value of $d = 4p_n +$ diameter of arbor.

Outside diameter of hob j
= d + 2 × dedendum of gear tooth.

Lead of hob thread $L = tp_n + \dfrac{tp_n}{2\left(\dfrac{\pi d}{tp_n}\right)^2 - 1{\cdot}5}$ (if $\dfrac{\pi d}{tp_n}$ exceeds 4)

or $tp_n \Big/ \sqrt{\left[1 - \left(\dfrac{tp_n}{\pi d}\right)^2\right]}$ (in general); $\tan\lambda = \dfrac{L}{\pi d}$

Tip radius of flute-cutter $= 0{\cdot}125\ p_n$

Lead of flutes $l = \dfrac{\pi^2 d^2}{L}$

Spiral angle of flutes $\sigma = \lambda$.

Number of flutes $F = \dfrac{\sqrt{[\pi^2 d^2 + L^2]}}{1{\cdot}3\ p_n}$
(taken to nearest whole number).

The number of flutes F and the number of threads t should preferably have no common factor.

Rise of relieving cam $=\dfrac{\sqrt{[\pi^2 j^2+L^2]}}{10\,F}$ (approx.).

Ratio of change-gears for relieving cam:

$$\frac{\text{Product of numbers of teeth in driving gears}}{\text{Product of numbers of teeth in driven gears}}$$

$=K\times\left(\dfrac{\text{Number of flutes}}{\text{number of lobes in cam}}\right)\dfrac{l+L}{l}$ where K is a constant for the machine.. The number of lobes in the cam is usually unity.

The Manufacture of End-Mills for Gear-Cutting.—The end-mill process is usually employed for producing gear teeth only when more accurate processes are impracticable, *e.g.* in the manufacture of triple-helical gears with continuous teeth.

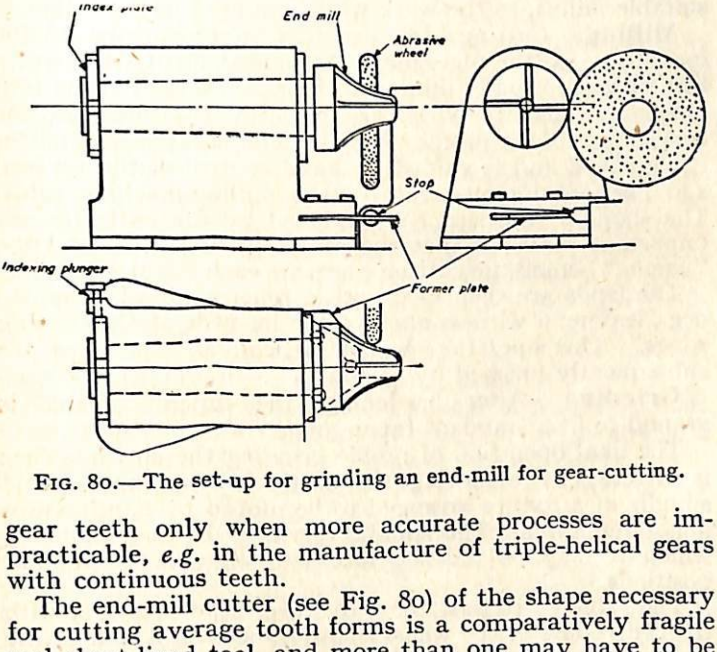


FIG. 80.—The set-up for grinding an end-mill for gear-cutting.

The end-mill cutter (see Fig. 80) of the shape necessary for cutting average tooth forms is a comparatively fragile and short-lived tool, and more than one may have to be used in completing a large gear. In the smaller sizes the cutter is made in one piece with the shank, which is tapered to fit into the spindle of the machine. In large sizes it is practicable to make cutter and shank as separate pieces, the one from high-speed tool steel and the other from plain carbon steel.

Turning.—The tapered shank is turned in the lathe and a tapped hole made for reception of a draw-bolt when the cutter is in use. The body of the cutter is turned to a profile which fits a " half-shape " gauge, the diameter at a specified distance from the end being measured by a vernier gear-tooth caliper, and made to leave an allowance of about 0·010 in. for grinding. The profiling is accomplished by feeding the cutting tool inwards by hand, whilst the saddle has a slow traverse away from the headstock ; the final shape is achieved by applying a hand-scraper, ground to a suitable radius, to the work whilst running in the lathe.

Milling.—Cutting edges are next produced by a milling operation. As the edges must continue almost to the centre-line they are usually limited to four ; in large cutters four extra intermediate edges are sometimes formed on the curved part of the profile. A plain cylindrical milling cutter is employed and is guided by hand control of the vertical and horizontal movements of the milling-machine table. The shape of the surfaces produced by the cutter is not important provided that the radial cutting faces and the " lands " behind the cutting edges are each about $\frac{1}{4}$ in. wide.

The lands are then filed with a relief angle of about 10 deg., leaving a witness about 0·020 in. wide at the cutting edges. This operation is not critical, as the edges are subsequently finished by grinding.

Grinding.—After hardening, the tapered shank is ground to fit a standard taper gauge.

The final operation of profile grinding the cutting edges is effected by mounting the cutter in the taper bored spindle of a fixture arranged to be moved by hand over a horizontal table. The spindle carries a dividing plate by which it may be indexed into four equidistant angular positions.

The table carries an abrasive wheel head arranged with the centre-line of the wheel slightly lower than that of the centre-line of the cutter. Vertically beneath the periphery of the abrasive wheel is a stop of the same shape as the plan view of the edge of the wheel.

A former plate is attached to the cutter fixture in such a position as to correspond with the desired profile and to bear on the fixed stop. Thus when the fixture is guided by hand so that the former plate always makes contact with the stop, the abrasive wheel produces an edge of the correct shape on a cutting face which lies in a horizontal plane. The difference in height of centre-lines of cutter and abrasive wheel is such as to give a suitable relief angle.

The grinding is continued equally on each cutting edge until the diameter at a certain distance from the end corresponds to the calculated figure. As a check on the shape of the profile, the diameter should be measured in two other planes as widely separated as possible and the figures compared with those obtained by calculation.

Determination of the Shape.—The shape of the end-mill required to produce any helical gear teeth of involute form can be determined by calculation. Except in the case of spur gears the end-mill shape is not involute, although it approximates to it if the helix angle is small. The shape may be taken as that corresponding to a spur gear having pitch and pressure angle equal to the normal pitch and normal pressure angle of the actual gear and a number of teeth equal to the actual number multiplied by the cube of the secant of the spiral angle. If this latter number exceeds about 60 the approximation is adequate for good-class commercial gears.

The cutter is re-sharpened when necessary by the method used for grinding its form in the first instance.

Gear-Generating Cutter.—The four principal methods practised for cutting gear teeth are: (1) by means of a rotary disc cutter formed to the tooth profile; (2) by means of an end-mill; (3) by means of a planer; and (4) by generation.

The rotary disc cutter has the disadvantage that practical considerations compel the use of the same cutter through a considerable range of sizes of gears, whereas to obtain a true tooth shape a cutter would be required for every size; in one well-known system only eight cutters are supplied to cut all gears of a given pitch from 12 teeth to a rack, which latter may be considered as a gear of infinite radius. For the second method, it may be said that it is chiefly used for herring-bone teeth, and would not be used except in special circumstances for straight-cut teeth.

The third practice is chiefly of a makeshift nature. The fourth, however, has now been accepted as the only correct method of cutting gears, and its introduction has made possible the cutting of gears of greater accuracy than by any other process.

Before dealing with the actual manufacture of gear cutters it is necessary to understand certain principles and functions relating to generation.

There are two principal methods of machining a contoured or other surface, namely, forming or generating. To illustrate this point, a surface plate may be planed by using a tool having a perfectly straight cutting edge, the full width of the plate, feeding the tool gradually at every stroke; this is an example of forming a surface.

On the other hand, if a tool is fed across the work, and moved slightly in a parallel plane at each successive stroke, the surface would be generated. Similarly a cylinder may be formed on the milling machine, or generated in the lathe. It is evident that the generating method of obtaining a surface is the more accurate, whether applied to gears or cylinders.

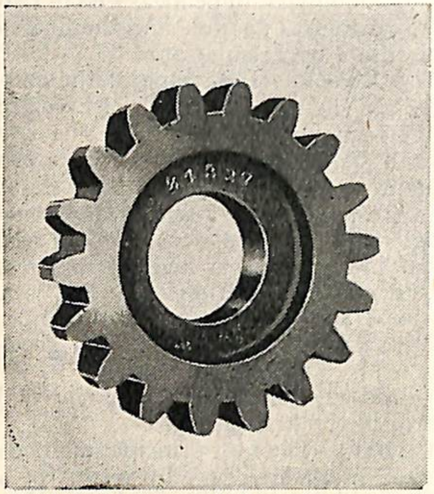
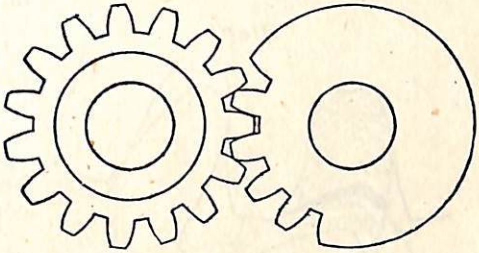


FIG. 81.—A typical gear cutter.

Gear Generation.—The generation of gears is made possible by the use of a cutter which is in itself a gear (see Fig. 81), and the action of the tool is clearly shown in Fig. 82. This illustrates a metal gear being rolled and in mesh with a material of plastic nature, such as wax. When rolled together as two gears in mesh, the generating gear will mould teeth in the plastic material of correct shape;

therefore, all gears so shaped will mesh with one another, or with their generator.

Fig. 83 shows this principle as applied to the cutting of metal blanks. The cutter is reciprocated vertically by the ram of the gear shaper on which it is mounted. During this motion the tool is first gradually fed inwards until the correct depth has been reached, then the cutter and the generated gear are revolved together exactly as two gears in mesh. The teeth of the revolving and reciprocating cutter thus form teeth of the correct form in the blank. Obviously, the teeth of the generating cutter must be extremely accurate, with proper clearance angles, in order to ensure free cutting. The sequence of the tool movements will be gathered by referring to Fig. 84.



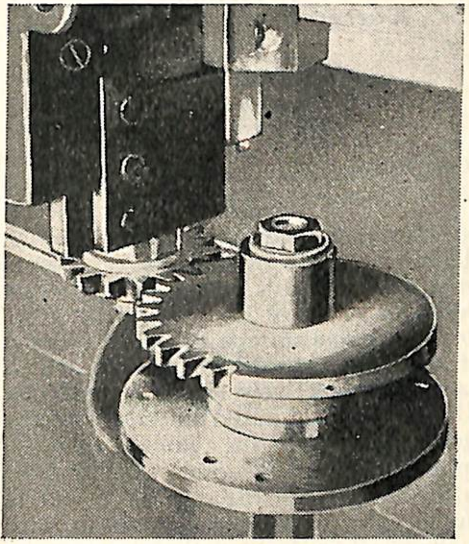


FIG. 82 (*above*).—A sketch showing the action of the tool illustrated in Fig. 81.

FIG. 83 (*left*).—The cutter at work on a metal blank.

Making the Cutter.—As is the practice with other gears, the cutter is firstly roughly machined, bored and faced on the turret lathe. The teeth are then profiled roughly either on the milling machine or on the gear shaper by means of cutters, which leave a sufficient tolerance on the tooth thickness for final grinding to shape.

After these preliminary roughing operations have been effected the cutter is hardened, and it will thus be seen that on account of the following finishing and final grinding operations there is no risk of the tool being inaccurate. As the accuracy of subsequent operations is dependent on the bore, this is finished first, after which, the front and back faces of the teeth and blank and any succeeding operation that may be necessary, are completed.

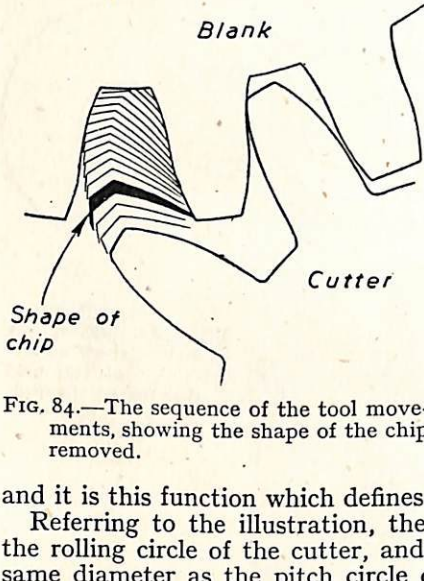


FIG. 84.—The sequence of the tool movements, showing the shape of the chip removed.

The Involute Grinder.—Figs. 85 and 86 outline the machine and principles for grinding the profile of the teeth to a true involute curve. This operation is effected by the generating process, and it is this function which defines a generated cutter.

Referring to the illustration, the cylinder A represents the rolling circle of the cutter, and it is, therefore, of the same diameter as the pitch circle of the generating gear. This cylinder oscillates on a plane surface B, and is ensured against slipping by the action of the steel tapes C, which are fixed to the cylinder and positively control the rolling motion. The cutter D (Fig. 85) is mounted on the end of the spindle, which is rigidly secured to the cylinder. As the cylinder oscillates up and down on the inclined plane the cutter is caused, by means of the steel tapes, to roll in exactly the same way as if it were a gear in mesh with the imaginary rack, F, as denoted by dotted lines in Fig. 88.

The straight face of the grinding wheel, E (Fig. 85), really represents one of the flat faces of the rack teeth, and since the rolling motion of the blank is controlled by the cylinder, together with the angular plane surface and steel tapes, it will cause the revolving grinding wheel to profile a true involute curve on the face of the cutter tooth or work. In order that one face only of each of the teeth of the cutter may be ground in turn, a dividing head is fitted so that the blank can be indexed in relation to the spindle. When this operation is completed the cutter is taken off and placed on a machine which is "left hand" to the first, and the same procedure is repeated on the opposite faces or sides of the teeth.

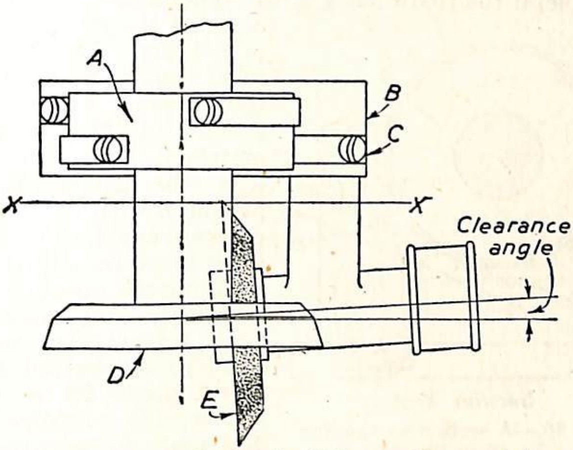


FIG. 85.—A diagrammatic sketch of the machine used for grinding the profile of the teeth to a true involute curve.

The face of the grinding wheel should be dressed to great accuracy by means of a diamond, which should be truly traversed across the face of the wheel.

A string being unwound from a cylinder describes the involute curve, and in this grinding operation the steel bands impart a similar movement; thus, since the cutter motion is synchronous with that of the rack the correct shape must necessarily be transferred to the tooth being ground.

Pressure Angle Corrections.—The head which carries the grinding wheel has a swivelling movement, so that the cutters for generating helical gears may be ground. It must, of course, have a relative motion in a right-angled direction which permits the relief angles to be ground on each tooth flank.

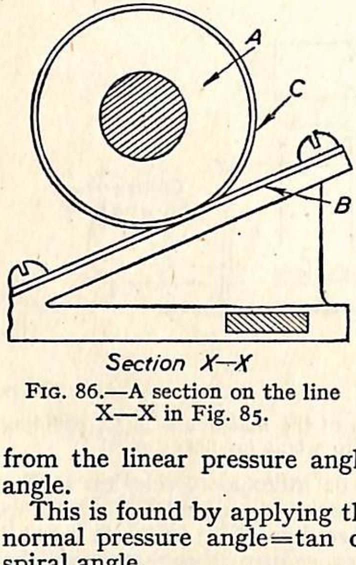


FIG. 86.—A section on the line X—X in Fig. 85.

With regard to relief angles, these are involved in the calculations for determining the pressure angle. For example, if the tooth has a 2 deg. relief angle, the pressure angle of the grinding wheel is $14\frac{1}{2}$ deg. However, the relief angle will increase the pressure angle of the cutter by 12′ approximately; thus, in order that the cutter may be produced with the required pressure angle, this discrepancy must be adjusted by inclining the grinding wheel head $14\frac{1}{2}^\circ - 12' = 14^\circ\ 18'$.

In setting the grinding head to correspond to the spiral angle for a helical cutter, the pressure angle has again to be corrected; it has to be transferred from the linear pressure angle into the normal pressure angle.

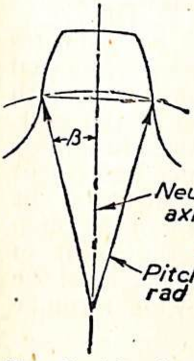
This is found by applying the following formula: tan of normal pressure angle = tan of linear pressure angle × cos spiral angle.

In checking the chordal thickness of a helical cutter, it is easier and more convenient to measure it normally to the spiral angle. The normal thickness (NT) = circular thickness × cos spiral angle.

It frequently happens that a cylinder, to which the steel tapes are fitted, required with a pitch diameter corresponding to that of the cutter desired to be produced, is not in stock. The cylinder having a diameter nearest to that figure required should be used and the pressure angle corrected to correspond. The following formula will give the corrected pressure angle: cos press. angle required × correct pitch radius divided by pitch radius of cylinder used.

The normal pitch = cos spiral angle × circular pitch.

The following pressure angle data will be of great assistance in the design of gear-generating cutters.



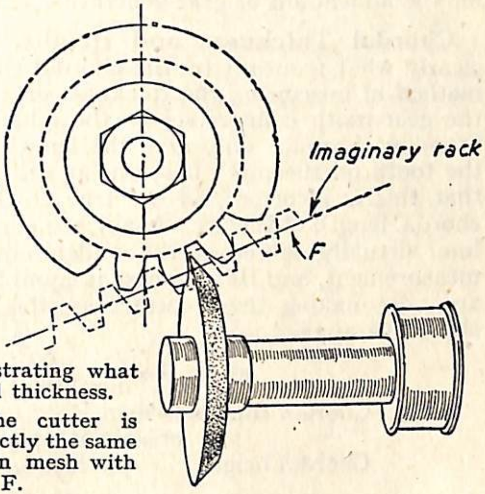


Fig. 87 (*above*).—Illustrating what is meant by chordal thickness.

Fig. 88 (*right*). — The cutter is caused to roll in exactly the same way as if it were in mesh with the imaginary rack F.

Tan normal pressure angle (NPA) = tan linear pressure angle (LPA) × cos spiral angle (SA).

$$\frac{\tan \text{NPA}}{\cos \text{SA}} = \tan \text{LPA}$$

$$\frac{\tan \text{NPA}}{\tan \text{LPA}} = \cos \text{SA}$$

$$\frac{\tan \text{LPA} \times \cos \text{SA}}{\tan \text{NPA}} = 1$$

$$\frac{\tan \text{NPA}}{\cos \text{SA} \times \tan \text{LPA}} = 1$$

Addendum of the Cutter.—In measuring the thickness of these cutters it should be remembered that the addendum of the cutter is the dedendum of the gear tooth ; calipers should therefore be set to a corrected addendum and not to that obtained by the formula,

$$\frac{1}{\text{diametral pitch (DP)}}$$

which only gives the addendum of the teeth cut. The rule for the addendum of gear-generating cutters is $\frac{1{\cdot}157}{\text{DP}}$.

Chordal Thickness and Height.—Fig. 87 indicates clearly what is meant by the chordal thickness—the usual method of measuring the thickness on the pitch line, with the gear-tooth calipers set to the addendum of the gear. Standard formulæ only give the length of the arc cut by the tooth on the pitch line, and it will at once be evident that this is incorrect, as the true thickness must be the chordal length of the arc. Again, the curvature of the pitch line virtually increases the addendum at the point of measurement, and this increase is equal to the height of the arc. In making these corrections the following formulæ should be applied :

$$\text{angle B} = \frac{90^\circ}{\text{number of teeth}}$$

$$\text{Chordal thickness} = \sin \text{B} \times 2 \text{ rad.}$$

$$\text{or} = \sin \text{B} \times \text{pitch diam.}$$

$$\text{Chordal height} = \text{pitch rad.} \times (1 - \cos \text{B})$$

CHAPTER X

LOAD CAPACITY OF GEARS

WHEN power is being transmitted by a pair of gears a force is exerted by each tooth on the one with which it makes contact, and the load capacity of the pair is determined by the ability of the teeth to resist the effects of such forces.

For example, the force P applied to the gear tooth shown in Fig. 89 causes local "surface stress" in the material near its point of application and also tensile and compressive "bending stresses" at A and B respectively. These stresses are applied and removed every time the tooth passes through the zone of contact, and the intermittency of the loading makes it more destructive than a constant loading of the same amount.

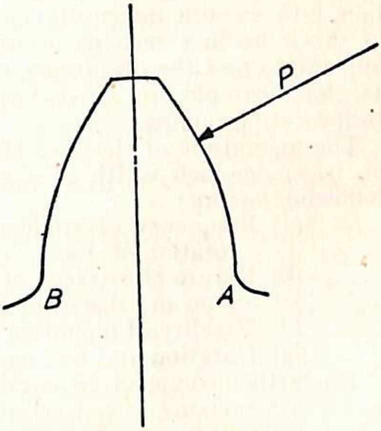


FIG. 89.—The force P causes intermittent tensile and compressive bending stresses at A and B.

Repetition of excessive surface stress usually leads to destruction of the tooth surface by "pitting," small pieces of metal breaking away to leave depressions of approximately spherical shape. If lubrication is inadequate, high surface pressure on the parts of the tooth profile which slide over the mating profile causes wear to occur in such parts. In extreme cases, the lubricant may be squeezed out, when sliding contact of metal on metal under heavy pressure may lead to partial welding together and "dragging" of the surfaces, leaving them too rough and inaccurate for smooth meshing.

In most cases the load capacity of a pair of gears is limited by surface pressure rather than by bending stress. In other words, the teeth are more likely to fail by destruction of the working surfaces than by breakage of the teeth. If the material is case-hardened steel, however, the reverse may be true, because the surface is much harder than that of steel in its normal unhardened condition, and so the teeth can withstand relatively high surface stress whilst their bending strength is not proportionately increased. Also, teeth made from a brittle material such as cast-iron may snap off under a shock load which, if only occasionally repeated, might not have had any serious effect on the working surfaces.

Shock Loading.—In designing gears it is necessary to take into account intermittency of loading, the possibility of shock loading such as occurs, for example, in rolling mill drives and the occurrence of continual load pulsation as, for example, in drives associated with engines or reciprocating pumps.

The magnitude of the load that may safely be imposed on (say) one-inch width of a gear tooth depends on the following factors:

(a) Frequency of application of load, *i.e.* speed of rotation of gear.
(b) Nature of material of gear.
(c) Shape and size of tooth.
(d) Rigidity of mounting of gear.
(e) Duration and frequency of loading periods.

No further comment is needed on (a) and (b). So far as surface pressure is concerned the important feature of (c) is the "relative radius of curvature" of the teeth at the line of contact. This varies to some extent according to the position of the line of contact on the profile, but its average value is used as a means of determining the allowable surface pressure. Broadly it may be said that for gears of given normal pressure angle and spiral angle, the permissible load per inch width is proportional to the 0·8th power of the pitch diameter of the pinion. The limiting figure in any particular instance may be calculated from the dimensions of the gears with the aid of formulæ published in the appropriate British Standard specification.

Bending Stress.—So far as bending stress is concerned, the important features of (c) are the depth of the tooth and its thickness at the root. These are controlled largely by the pitch, but even for a given pitch the tooth thickness varies somewhat according to the number of teeth. Here again the British Standard specification indicates how to determine load capacity on the basis of "strength" (as this is conveniently described in distinction from "wear") of any pair of gears.

It may be observed that if the materials, diameters and width of a pair of gears are fixed, the load capacity on the basis of "strength" may be varied by varying the numbers of teeth, or, in other words, varying the pitch. Such a change, however, has no appreciable effect on the load capacity on the basis of "wear," and as this is usually the limiting factor it follows that nothing is to be gained by adopting excessively large pitches.

Smoothness of running is improved by the use of a fine pitch, and as a general rule the best design practice is to use the finest pitch that will give adequate strength. Except when case-hardened steel is used this means that the most suitable number of teeth in the pinion is rarely less than about 25.

Working Life of a Gear.—The rigidity of mounting of the gears affects the load capacity, because it has an influence on the distribution of loading across the face of the gear. It is not easy, or usual, to attempt any numerical assessment of this factor, and indeed its variation between different examples is not great if care is taken in design to provide a mounting of the utmost practicable rigidity.

The standard basis of gear design gives a normal life-expectation of 25,000 hours, or, roughly, 2,000 working days of 12 hours each. If the operating time in any particular instance is known to be different from 12 hours per day, it is usual to modify the working stresses so as to keep the normal expectation of life at 2,000 working days. The probable life is approximately proportional to the reciprocal of the third power of the working stress; for example, if the required length of life were only one-eighth of 25,000 hours the working stresses might be double those applying to the standard 25,000-hour rating.

In some instances, particularly in the case of worm gears, continuous operation may lead to such a rise of temperature (due to generation of heat by friction and oil turbulence) that the oil tends to lose its lubricating qualities, when rapid wear of the teeth commences. It can therefore happen that the limit of load that can be safely imposed on gears is set by temperature-rise rather than by stresses in the teeth.

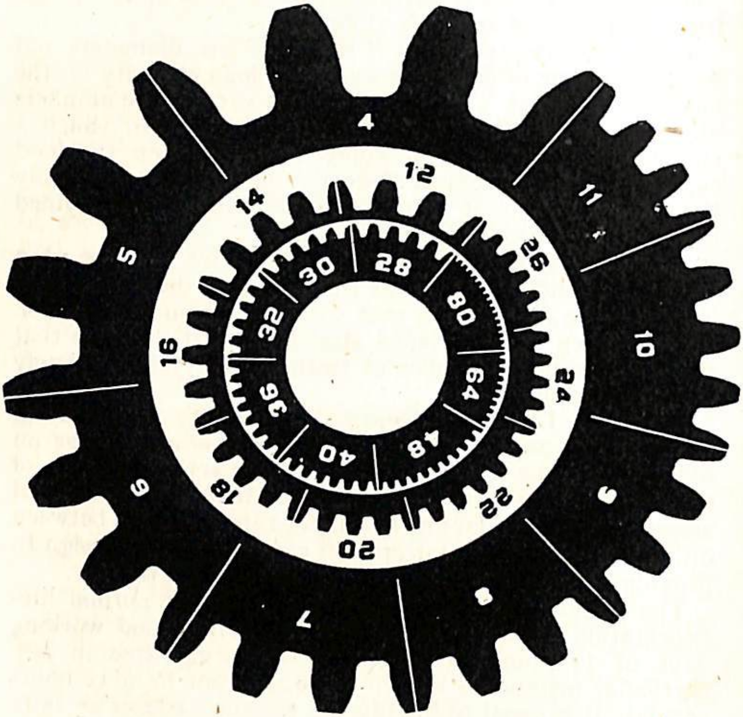


FIG. 89A.—Comparative Sizes of Involute Gear Teeth.

Worm gears are especially prone to this limitation because of the relatively great amount of sliding associated with their tooth action, whilst high-speed gears of any type are liable to encounter it by reason of power loss in oil-churning.

In order to raise this limitation, recourse may be had to artificial cooling devices in the form of fans or cooling coils. Alternatively, the oil may be sprayed on to the gears in the zone of contact (thus avoiding the considerable power loss otherwise caused by turbulence in the oil-bath), drained from the bottom of the gearcase, and passed through an oil-cooler before being returned to the gears.

The accepted formulæ relating to the strength and load capacity of gears are published in British Standard Specifications Nos. 436, 545 and 721. These have been prepared in collaboration with the leading gear manufacturers. They are obtainable from the British Standards Institution, 28 Victoria Street, London, S.W.1.

CHAPTER XI

THE EFFICIENCY OF GEARS

THE amount of power lost by friction between the teeth of most gears is usually less than 1 per cent. of the transmitted power, but in estimating the efficiency of the whole gear drive it is necessary to take into account the power lost in bearing friction, oil-drag, and windage. Bearing friction may absorb only about the same amount of power as tooth friction, but oil-drag may be more serious, particularly if the peripheral speed is high. Windage is unimportant except at very high speeds.

The overall efficiency of a single-stage gear assembly of any of the types mentioned above may be expected to lie between 97 and 98 per cent. at full load under favourable conditions. At fractional loads the efficiency is lower, largely because oil-drag (on the teeth and in the bearings) does not diminish as the load drops and therefore becomes a greater fraction of the transmitted power.

The power lost by friction between the teeth of worm gears depends on the lead angle of the worm, and therefore indirectly on the reduction ratio. In this type of gear there is a greater amount of relative sliding than in other forms, and consequently tooth efficiency is lower. Oil-drag is also a cause of appreciable loss of power if the worm-shaft speed is high. To take all the factors into account is difficult, but as an approximate guide the overall efficiency of a worm-gear unit with a worm-shaft speed not higher than about 1000 r.p.m. may be taken as equal to 100 minus half the reduction ratio, *e.g.* 30 to 1 would give 85 per cent. This assumes the use of a case-hardened and ground steel worm in conjunction with a phosphor-bronze worm-wheel. At higher worm-shaft speeds the efficiency tends to be slightly lower because of greater oil-drag. At worm-shaft speeds below about 100 r.p.m the efficiency again tends to be lower because of the rise in co-efficient of friction at low rubbing speeds.

Lubrication of Gears.—Gears which run only very intermittently may be lubricated by grease, but in general oil lubrication is essential if the full load-capacity of the gear-materials is to be realised. Oil of high viscosity is necessary when the tooth-loading is heavy, but is unsuitable if the speed is high because, on the one hand, it may lead to serious power loss by churning, or may " channel " to such an extent under the action of the gears that they are not satisfactorily lubricated. Fortunately, consideration of fatigue makes it impracticable to subject high-speed gears to very heavy loading, and it is therefore possible to lubricate them with the low-viscosity oil which is necessary to keep oil-drag loss within reasonable bounds.

Up to peripheral speeds of about 2500 f.p.m., lubrication of gears is adequately effected by allowing one of a pair to dip into an oil-bath. At higher speeds oil-drag loss tends to become serious, and on this account it is usual to employ an oil-spray, keeping the gears clear of the oil in the sump.

The value of a particular oil as a gear lubricant is not shown by viscosity alone, and it is necessary to make sure that the oil used for any important gears carries the approval of the gear manufacturer. This is especially important in the case of worm gears because they are liable to complete failure in a very short time if lubrication is defective, whereas the other conventional types of gear are rather less sensitive in this respect.

CHAPTER XII

USEFUL FORMULÆ FOR GEARS

Spur Gearing.—The Standard gear terms and symbols, forming the basis of calculations concerning their tooth proportions, are shown in Figs. 90 and 91. It should be noted that *circular pitch* refers to the distance, measured round the pitch line, from the centre of one tooth to the centre of the next tooth; whereas *diametral pitch* refers to the number of teeth per inch of the pitch diameter. It is usual to refer to gears according to their diametral pitch.

Module is found by dividing the pitch diameter by the number of teeth. Thus a 40-tooth gear with 20-in. pitch diameter would have 2 teeth per inch of its pitch diameter and would thus be:

$$\frac{40}{20}=2\text{DP}.$$

The module of the same gear would be:

$$\frac{20}{40}=0{\cdot}5.$$

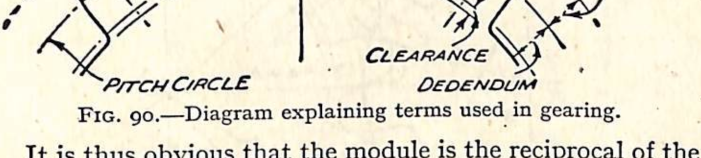


FIG. 90.—Diagram explaining terms used in gearing.

It is thus obvious that the module is the reciprocal of the diametral pitch, and *vice-versa*. The following formulæ apply to spur gears, where the addendum is equal to 0·3183 × circular pitch:

t=number of teeth.
a=addendum or module.
b=dedendum.

g=thickness of tooth on pitch line.
g''=chordal thickness of tooth.
c=clearance at bottom of tooth.
j=outside diameter.
d=pitch diameter.
z=working depth of tooth.
$a+b$=whole depth of tooth.
i=bottom diameter.
P=diametral pitch.
p=circular pitch.
H=height of arc.
s''=distance from chord of pitch line to top of tooth.
$\theta=\frac{1}{4}$ angle subtended by circular pitch.

The following formulæ apply to the system in which the addendum is equal to 0·3183 × circular pitch.

$$\mathrm{P}=\frac{t+2}{j};\ \text{or } \frac{\pi}{p}.$$

$$p=\frac{\pi}{\mathrm{P}};\ \text{or } d\pi\,\frac{\theta}{90};\ \text{or } \frac{d\pi}{t};\ \text{or } \frac{j\pi}{t+2}.$$

$$a=\frac{1}{\mathrm{P}};\ \text{or } \frac{p}{\pi};\ \text{or } 0{\cdot}3183\,p;\ \text{or } \frac{d}{t};\ \text{or } \frac{j}{t+2}.$$

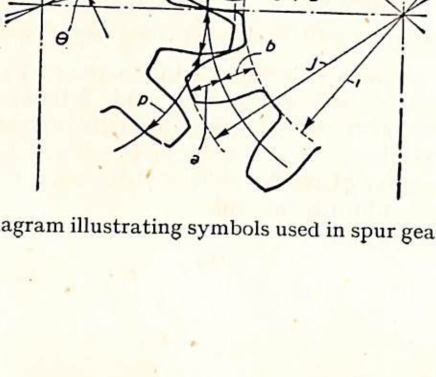


FIG. 91.—Diagram illustrating symbols used in spur gearing formulæ

$$g=\frac{p}{2}\text{; or }\frac{\pi}{2P}\text{; or }\frac{1{\cdot}5708}{P}.$$

$$c=0{\cdot}05\,p.$$

$$b=\frac{1{\cdot}157}{P}\text{; or }0{\cdot}3683\,p.$$

$$z=2a\text{; or }\frac{2}{P}.$$

$$a+b=\frac{2{\cdot}157}{P}\text{; or }0{\cdot}6866\,p.$$

$$d=\frac{t}{P}\text{; or }\frac{tp}{\pi}\text{; or }0{\cdot}3183\,tp\text{; or }\frac{jt}{t+2}.$$

$$j=d+2a\text{; or }\frac{t+2}{P}.$$

$$\theta=\frac{90^\circ}{t}.$$

$$g''=d\sin\theta.$$

$$i=j-2(a+b)\text{; or }\frac{t-2{\cdot}31416}{P}.$$

$$t=dP\text{; or }jP-2.$$

$$H=d\sin^2\frac{\theta}{2}\text{; }s''=a+H.$$

B. & S. Involute Gear Cutters

No.	1	will	cut	wheels	from	135	teeth	to	a rack		inclusive.
,,	2	,,	,,	,,	,,	55	,,	,,	134	teeth	,,
,,	3	,,	,,	,,	,,	35	,,	,,	54	,,	,,
,,	4	,,	,,	,,	,,	26	,,	,,	34	,,	,,
,,	5	,,	,,	,,	,,	21	,,	,,	25	,,	,,
,,	6	,,	,,	,,	,,	17	,,	,,	20	,,	,,
,,	7	,,	,,	,,	,,	14	,,	,,	16	,,	,,
,,	8	,,	,,	,,	,,	12	,,	,,	13	,,	,,

Bevel Gears

(Axes at Right Angles)

T=Number of teeth on gear.
t=Number of teeth on pinion.
P=diametral pitch.
p=circular pitch.

θ_w=pitch angle of gear.
θ_p=pitch angle of pinion.
ϕ_w=angle of face of gear.
ϕ_p=angle of face of pinion.
ρ_w=cutting angle of gear.
ρ_p=cutting angle of pinion.
C=apex distance from pitch circle.
j=outside diameter of pinion.
J=outside diameter of wheel.
d=pitch diameter of pinion.
D=pitch diameter of wheel.
a=working depth of tooth.
z=addendum or module.
b=dedendum.
g=thickness of tooth at pitch line.
c=clearance at bottom of tooth.
$a+b$=whole depth of tooth.
F=width of face.

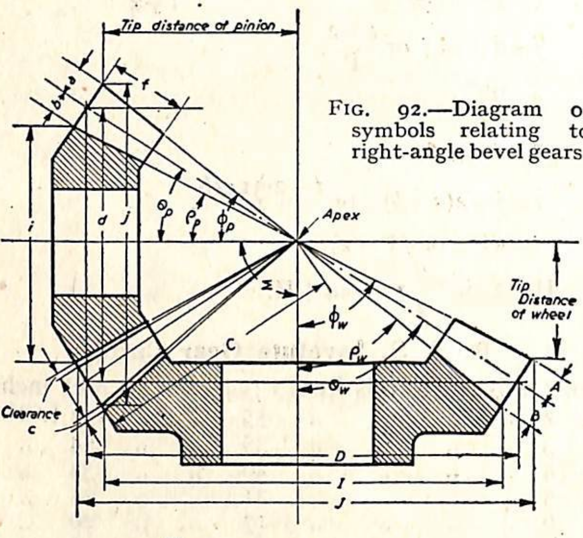

Tip distance of pinion
f
b+a
i
d
j
Θp
ρp
φp
Apex
Σ
φw
ρw
Θw
C
Tip Distance of wheel
A
B
Clearance c
D
I
J

FIG. 92.—Diagram of symbols relating to right-angle bevel gears.

$$\tan \theta_w = \frac{\mathrm{T}}{t}; \ \tan \theta_p = \frac{t}{\mathrm{T}}.$$

$$a = \frac{1}{\mathrm{P}}, \text{ or } 0{\cdot}3183\, p. \qquad b = \frac{1{\cdot}157}{\mathrm{P}}, \text{ or } 0{\cdot}3683\, p.$$

$$\phi_p = \theta_p + \left(3440\, \frac{a}{\mathrm{C}}\right) \text{ minutes.}$$

$$\phi_w = \theta_w + \left(3440\, \frac{\mathrm{A}}{c}\right) \text{ minutes.}$$

$$\rho_w = \theta_w - \left(3440\,\frac{B}{C}\right) \text{ minutes.}$$

$$\rho_p = \theta_p - \left(3440\,\frac{b}{C}\right) \text{ minutes.}$$

$$d = \frac{tp}{\pi}, \text{ or } \frac{t}{P}, \text{ or } \frac{jt}{t + 2\cos\theta_p}.$$

$$D = \frac{T_p}{\pi}, \text{ or } \frac{T}{P}, \text{ or } \frac{JT}{T + 2\cos\theta_w}.$$

$$j = d + 2a\cos\theta_p.$$

$$J = D + 2a\cos\theta_w.$$

$$C = \frac{1}{2P}\sqrt{(T^2 + t^2)}.$$

$$C = \frac{D}{2}\operatorname{cosec}\theta_w = \frac{d}{2}\operatorname{cosec}\theta_p.$$

$$P = \frac{T}{2C}\operatorname{cosec}\theta_w, \text{ or } \frac{T + 2\cos\theta_w}{J}.$$

$$p = \frac{\pi}{P}.$$

CUTTERS FOR MITRE AND BEVEL GEARS

Diametral Pitch	*Diameter of Cutter*	*Hole in Cutter*
4	3½ in.	1¼ in.
5	3½ in.	1¼ in.
6	3½ in.	1¼ in.
7	3½ in.	1¼ in.
8	3¼ in.	1¼ in.
10	3¼ in.	⅞ in.
12	3	⅞ in.
14	3	⅞ in.
16	2¾ in.	⅞ in.
20	2½ in.	⅞ in.
24	2¼ in.	⅞ in.

TABLE FOR OBTAINING SET-OVER FOR CUTTING BEVEL GEARS

No. of Cutter	Ratio of apex distance to width of face $\left(\frac{C}{F}\right)$												
	$\frac{3}{1}$	$\frac{3\frac{1}{4}}{1}$	$\frac{3\frac{1}{2}}{1}$	$\frac{3\frac{3}{4}}{1}$	$\frac{4}{1}$	$\frac{4\frac{1}{4}}{1}$	$\frac{4\frac{1}{2}}{1}$	$\frac{4\frac{3}{4}}{1}$	$\frac{5}{1}$	$\frac{5\frac{1}{2}}{1}$	$\frac{6}{1}$	$\frac{7}{1}$	$\frac{8}{1}$
1	0·254	0·254	0·255	0·256	0·257	0·257	0·257	0·258	0·258	0·259	0·260	0·262	0·264
2	0·266	0·268	0·271	0·272	0·273	0·274	0·274	0·275	0·277	0·279	0·280	0·283	0·284
3	0·266	0·268	0·271	0·273	0·275	0·278	0·280	0·282	0·283	0·286	0·287	0·290	0·292
4	0·275	0·280	0·285	0·287	0·291	0·293	0·296	0·298	0·298	0·302	0·305	0·308	0·311
5	0·280	0·285	0·290	0·293	0·295	0·296	0·298	0·300	0·302	0·307	0·309	0·313	0·315
6	0·311	0·318	0·323	0·328	0·330	0·334	0·337	0·340	0·343	0·348	0·352	0·356	0·362
7	0·289	0·298	0·308	0·316	0·324	0·329	0·334	0·338	0·343	0·350	0·360	0·370	0·376
8	0·275	0·286	0·296	0·309	0·319	0·331	0·338	0·344	0·352	0·361	0·368	0·380	0·386

$$\text{Set-Over} = \frac{Tc}{2} - \frac{\text{Factor from Table.}}{P}$$

Tc = thickness of cutter used.
P = diametral pitch of gear to be cut.

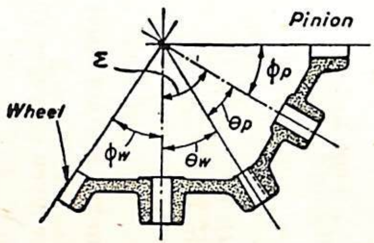

Pinion
Σ
Wheel
ϕ_P
θ_P
ϕ_W
θ_W

FIG. 93.

Relevant formulæ are on next page.

FIGS. 93 to 96.—Four cases of bevel gears with axes at angles greater and less than 90°.

Definition of symbols used in these diagrams appears on page 126.

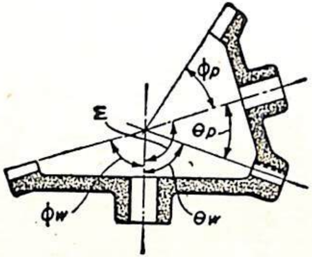

Σ
ϕ_P
θ_P
ϕ_W
θ_W

FIG. 94.

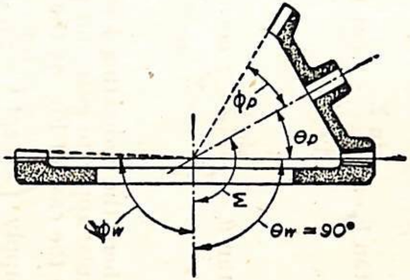

ϕ_P
θ_P
Σ
ϕ_W
$\theta_W = 90°$

FIG. 95.

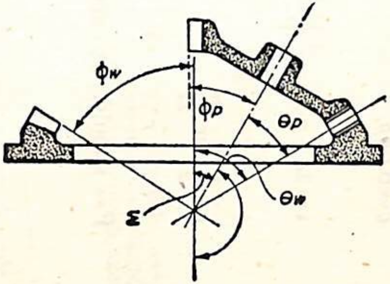

ϕ_W
ϕ_P
θ_P
θ_W
Σ

FIG. 96.

$c=0.05p.$

$z=2a$; $z+c=\frac{2{\cdot}157}{P}$, or $0{\cdot}6866\ p$.

$g=\frac{p}{2}$, or $\frac{\pi}{2P}$.

$F=$ should not exceed $\frac{C}{3}$.

Bevel Gears

(Axes at any Angle)

$\Sigma=$angle formed by axes of gears.

The formulæ for tooth parts on pages 121 to 124 also apply here.

$$\tan\theta_p=\frac{\sin\Sigma}{\frac{T}{t}+\cos\Sigma};\quad \cot\theta_p=\frac{T}{t\sin\Sigma}+\cot\Sigma.$$

$$\tan\theta_w=\frac{\sin\Sigma}{\frac{t}{T}+\cos\Sigma};\quad \cot\theta_w=\frac{t}{T\sin\Sigma}+\cot\Sigma.$$

Where the indicating letters p and w are not used the formulæ apply to gear and pinion.

In the case of Fig. 93 the formulæ given on pages 124 to 126 apply $\left(\text{except } C=\frac{1}{2P}\sqrt{(T^2+t^2)}\right)$.

In the case shown by Fig. 95:

$$\theta_w = 90^\circ;\ \theta_p = \Sigma - 90^\circ.$$

In the case shown by Fig. 96:

$$\tan\theta_p = \frac{\sin\Sigma}{\frac{T}{t} - \cos\Sigma};\ \tan\theta_w = \frac{\sin\Sigma}{\cos\Sigma - \frac{t}{T}}.$$

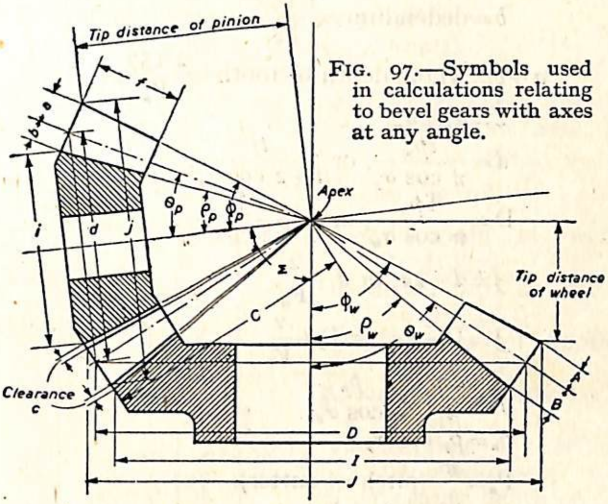

Tip distance of pinion
f
a
b
Apex
θ_p
ρ_p
ϕ_p
i
d
j
Σ
C
ϕ_w
ρ_w
θ_w
Tip distance of wheel
A
B
Clearance c
D
I
J

FIG. 97.—Symbols used in calculations relating to bevel gears with axes at any angle.

Spiral or Screw Gearing

$\left.\begin{matrix} t \\ T \end{matrix}\right\}$=Number of teeth in gears.

C=Centre distance.

p=circular pitch.

P_n=normal diametral pitch.

p_n=normal pitch.

Σ=angle between axes.

L=lead of tooth helix.

$\left.\begin{matrix} t_e \\ T_e \end{matrix}\right\}$=number of teeth marked on cutter.

$d=$ pitch diameter.

$j=$ outside diameter.

$\left.\begin{matrix}\sigma_p\\ \sigma_w\end{matrix}\right\}=$ angle of teeth with axis.

$g=$ thickness of tooth.

$a=\text{addendum}=\dfrac{1}{P_n}.$

$b=\text{dedendum}=\dfrac{1{\cdot}157}{P_n}.$

$a+b=\text{whole depth of tooth}=\dfrac{2{\cdot}157}{P_n}.$

$$\Sigma=\sigma_p+\sigma_w.$$

$$d=\frac{tp_n}{\pi\cos\sigma_p},\ \text{or}\ \frac{tj}{t+2\cos\sigma_p}.$$

$$D=\frac{Tp_n}{\pi\cos\sigma_w},\ \text{or}\ \frac{TJ}{T+2\cos\sigma_w}.$$

$$j=d+2a,\ \text{or}\ d+\frac{2}{P_n}.$$

$$J=D+2a,\ \text{or}\ D+\frac{2}{P_n}.$$

$$p=\frac{d\pi}{t},\ \text{or}\ \frac{p_n}{\cos\sigma_p}.$$

$$p_n=p_p\cos\sigma_p.$$

$$P_n=\frac{\pi}{p_n}\ \text{(pitch of cutter)}.$$

$$a=\frac{p_n}{\pi},\ \text{or}\ \frac{1}{P_n},\ \text{or}\ \frac{j-d}{2}.$$

$$g=\frac{1}{2}p_n.$$

$$t_c=\frac{t}{\cos^3\sigma_p}. \qquad T_c=\frac{T}{\cos^3\sigma_w}.$$

$$L_p=\frac{tp_n}{\sin\sigma_p}. \qquad L_w=\frac{Tp_n}{\sin\sigma_w}.$$

In many small shops the Brown & Sharpe system of milling involute gear teeth is still used. In this system a series of eight cutters are required to cut all gears having 12 or more teeth in any one pitch. The list of eight cutters supplied for each pitch appears on page 121.

Worm and Worm Wheels

L=lead of worm.
T=Number of teeth in gear.
j=outside diameter of worm.
d=pitch diameter of worm.
d''=diameter of hob.
G=throat diameter.
D=pitch diameter of worm-wheel.
J=blank diameter (to sharp corners).
C=distance between centres.
p=circular pitch for worm wheels or axial pitch for worms.
$\left.\begin{matrix} r' \\ r'' \end{matrix}\right\}$=see Fig. 98.
a=addendum of worm=$0{\cdot}3183p$.
b=dedendum of worm=$0{\cdot}3683p$.
g=thickness of tooth at pitch line.
g_n=normal thickness of tooth.
c=clearance at bottom of tooth=$0{\cdot}05p$.
$2a$=Working depth of tooth.

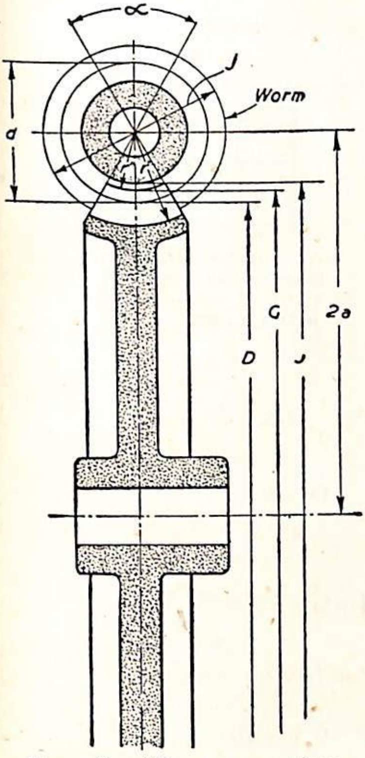


FIG. 98.—Worm proportions and symbols.

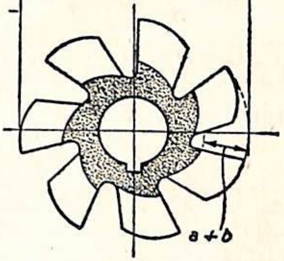


FIG. 99.—Hob proportions.

$a+b$=whole depth of tooth.

v=width of worm thread tool at end.

w=width of worm thread at tip and width of hob tool at end.

δ=angle of tooth of worm-wheel with its axis.

F=min. length of worm$=4p$.

F_1=length of hob$=6p$.

If the lead is for single, double or multiple start threads, $L=p, 2p, 3p$, etc.

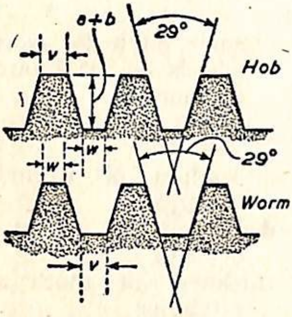


FIG. 100.—Diagram illustrating hob and worm relations.

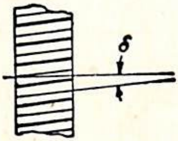


FIG. 101. — Diagram illustrating angle of tooth of worm-wheel with its axis.

$\alpha=60°$ to $90°$.

$$p=\frac{\pi G}{T+2}.$$

$$D=\frac{Tp}{\pi}.$$

$$G=D+2a.$$

$$\tan\delta=\frac{L}{\pi d}.$$

$$g_n=\frac{1}{2}p\cos\delta.$$

$$r'=\frac{d}{2}-a.$$

$$r''=\frac{1}{2}j+c.$$

$$C=\frac{D+d}{2}.$$

$$J=G+4r'\sin^2\frac{\alpha}{4}.$$

$$d''=d+2c.$$

$$v=0{\cdot}3095p.$$

$$w=0{\cdot}3354p.$$

Chordal Thickness of Teeth of Gears

(Basis of 1 Diametral Pitch)

t=Number of teeth in gear.
g''=Chordal thickness of tooth.
H=Height of arc.
d=Pitch diameter.
β=90° divided by the number of teeth.
$t''=d_1 \sin \beta$.
$H=d \sin^2 \frac{\beta}{2}$.

CHORDAL THICKNESS OF TEETH OF GEARS

(INVOLUTE)

Cutter	g	H	s'' $=a+H$
No. 1—135 T—1 P	1·5707	0·0047	1·0047
No. 2— 55 T—1 P	1·5706	0·0112	1·0112
No. 3— 35 T—1 P	1·5702	0·0176	1·0176
No. 4— 26 T—1 P	1·5698	0·0237	1·0237
No. 5— 21 T—1 P	1·5694	0·0294	1·0294
No. 6— 17 T—1 P	1·5686	0·0362	1·0362
No. 7— 14 T—1 P	1·5675	0·0440	1·0440
No. 8— 12 T—1 P	1·5663	0·0514	1·0514
11 T—1 P	1·5654	0·0559	1·0559
10 T—1 P	1·5643	0·0616	1·0616
9 T—1 P	1·5628	0·0684	1·0684
8 T—1 P	1·5607	0·0769	1·0769

Velocity Ratios of Gear Trains.—If all the gears in the train rotate about fixed centre lines, then

$$\text{Velocity ratio} = \frac{\text{Product of numbers of teeth in all driven gears}}{\text{Product of numbers of teeth in all driving gears}}$$

and

Revolutions of first driving gear= Velocity ratio × Revolutions of last driven gear.

Sprocket Wheels.—Chain or sprocket wheels are gears made to mesh with chains, by means of which motion may be transmitted from one point to another. The teeth are of different formation from the ordinary involute gear, as they have to accommodate the circular rollers of the chain. Ordinary gearing calculations for pitch line functions do not apply, for whereas with two gears in mesh with one another the circular pitch is the distance between two tooth centres measured round the pitch line, with a sprocket the circular pitch refers really to the chordal distance between two tooth centres. As information relating to these calculations is extremely scanty, the following formulæ for block-centre and roller-chain may be of interest.

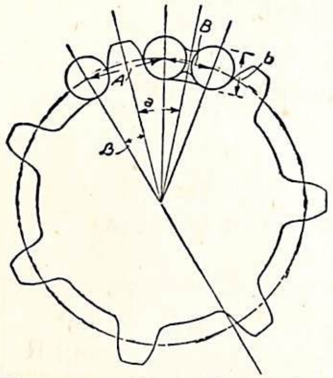


FIG. 102.—Proportions of a sprocket wheel on block-centre chains.

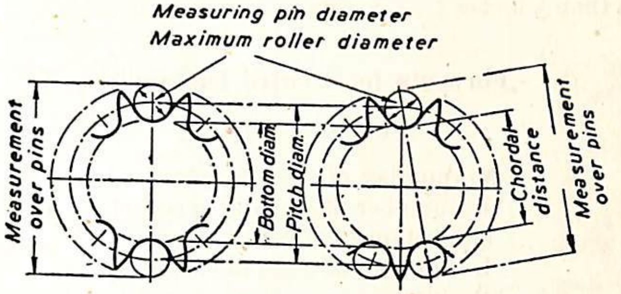
Sprocket Wheels for Block-Centre Chains.—Fig. 102 shows a wheel as used for block-centre chains, and the following formulæ give proportions and diameters relevant to them, $a=\frac{180^\circ}{N}$. $\tan\beta=\dfrac{\sin a}{\frac{B}{A}+\cos a}$.

$$\text{Pitch diam.}=\frac{A}{\sin\beta}.$$

Outside diam.=pitch diam.$+b$. Bottom diam.=pitch diam. $-b$.

In calculating the diameter of sprocket wheels the bottom diameter is most important. In the above calculations N=number of teeth, b=diameter of round part of chain block, B=centre to centre of holes in chain block, A=centre to centre of holes in side-links.

British Standard Roller Chain Wheels


Measuring pin diameter
Maximum roller diameter
Measurement over pins
Bottom diam.
Pitch diam.
Chordal distance
Measurement over pins

$$\text{Pitch Diameter} = \text{Chain pitch} \times \text{cosec.} \left(\frac{180^\circ}{\text{No. of teeth}}\right).$$

Bottom diameter = Pitch diameter minus roller diameter.

Chordal distance = [$\frac{1}{2}$ (pitch diameter of wheel having twice the number of teeth) minus roller diameter].

MEASUREMENT OVER PINS

The bottom diameter of wheels with *even* numbers of teeth is checked by measuring over pins inserted in opposite tooth spaces.

The bottom diameter of wheels with *odd* numbers of teeth is checked from chordal distance by measuring over pins inserted in the tooth spaces most nearly opposite.

Measurement over pins.
- For *even* numbers of teeth = pitch diameter plus roller diameter.
- For *odd* numbers of teeth = [$\frac{1}{2}$ (pitch diameter of wheel having twice the number of teeth) plus roller diameter].

Tolerance on Cutting Sizes—

BOTTOM DIAMETER AND MEASUREMENT OVER PINS

Plus 0·000 inch.

Minus 0·004 inch per inch (mm. per mm.) of pitch diameter, with a maximum of 0·020 inch (0·51 mm.), subject to the provision that a tolerance up to 0·005 inch (0·13 mm.) shall be permitted on all chain wheels with pitch diameter less than 5 inches (127·00 mm.).

Formula for Centre Distances

(Two point drives)

N=number of teeth in wheel.
n=number of teeth in sprocket.
D=distance between centres.
L=number of links in chain.
P=pitch.

$$L=\left[\frac{2\times D}{P}+\frac{N+n}{2}+\frac{P\times(N-n)^2}{40D}\right]P \text{ (approx).}$$

The result obtained from this expression will not ordinarily be a whole number of links ; in that event the nearest even number should be taken and a recalculation made to find the corresponding centre distance from the following :

$$D=\frac{P}{8}\left[A+\sqrt{(A+0{\cdot}9B)\times(A-0{\cdot}9B)}\,\right]$$

where $A=2L-(N+n)$ and $B=(N-n)$.

This formula gives results correct to within 0·05 per cent. for wheels having a ratio of 2:1 at minimum centres. This is sufficiently accurate for drives where adjustment is provided.

BLOCK-CENTRE CHAINS

Circular pitch	Thickness of block	Diameter of cutter	Centre to centre of block	Hole in cutter
In.	In.	In.	In.	In.
1 5/16	·4375	3 1/2	·5313	1 1/4
1 1/2	17/32	3 3/4	·5625	1 1/4

Seven cutters are made for each pitch, for Numbers of teeth as follows: 8, 9, 10, 11, 12, and 13, 14 to 16, 17 to 20, 21 and over.

ROLLER CHAINS

Circular pitch	Diameter of rolls	Diameter of cutter	Hole in cutter
In.	In.	In.	In.
$\frac{1}{2}$	·306 or ·308	$2\frac{7}{8}$	1
$\frac{5}{8}$	·401	3	1
$\frac{3}{4}$	*·47	$3\frac{1}{4}$	1
$\frac{15}{16}$	·5625	$3\frac{3}{8}$	1
1	·5625 or *·625	$3\frac{1}{2}$	1
$1\frac{1}{4}$	·625 or *·750	$3\frac{3}{4}$	$1\frac{1}{4}$
$1\frac{1}{2}$	·75 or *·875	$4\frac{1}{2}$	$1\frac{1}{4}$
$1\frac{3}{4}$	*1	$4\frac{3}{4}$	$1\frac{1}{4}$
2	*1·125	5	$1\frac{1}{4}$

* "Whitney Standard."

CIRCULAR PITCH

Circular Pitch is the Distance from the Centre of one Tooth to the Centre of the next Tooth, measuring along the Pitch Circle.

To Find	*Having*	*Rule.*	*Formula.*
The Circular Pitch	The Diametral Pitch	Divide 3·1416 by the Diametral Pitch	$p=\frac{3{\cdot}1416}{P}$
The Circular Pitch	The Pitch Diameter and Number of Teeth	Divide Pitch Diameter by the product of 0·3183 and Number of Teeth	$p=\frac{d}{0{\cdot}3183t}$
The Circular Pitch	The Outside Diameter and Number of Teeth	Divide Outside Diameter by the product of 0·3183 and Number of Teeth plus 2	$p=\frac{j}{0{\cdot}3183(t+2)}$
Pitch Diameter	Number of Teeth and the Circular Pitch	The continued product of the Number of Teeth, the Circular Pitch and 0·3183	$d=0{\cdot}3183tp$
Pitch Diameter	The Number of Teeth and the Outside Diameter	Divide the product of Number of Teeth and Outside Diameter by Number of Teeth plus 2	$d=\frac{tj}{t+2}$
Pitch Diameter	The Outside Diameter and Circular Pitch	Subtract from the Outside Diameter the product of the Circular Pitch and 0·6366	$d=j-0{\cdot}6366p$
Pitch Diameter	Addendum and the Number of Teeth	Multiply the Number of Teeth by the Addendum	$d=ta$

Outside Diameter	Number of Teeth and the Circular Pitch	The continued product of the Number of Teeth plus 2, the Circular Pitch and 0·3183	$j=0{\cdot}3183(t+2)p$
Outside Diameter	The Pitch Diameter and Circular Pitch	Add to the Pitch Diameter the product of the Circular Pitch and 0·6366	$j=d+0{\cdot}6366p$
Outside Diameter	The Number of Teeth and the Addendum	Multiply Addendum by Number of Teeth plus 2	$j=a(t+2)$
Number of Teeth	The Pitch Diameter and Circular Pitch	Divide the product of Pitch Diameter and 3·1416 by the Circular Pitch	$t=\frac{3{\cdot}1416d}{p}$
Thickness of Tooth	The Circular Pitch	One-half the Circular Pitch	$g=\frac{p}{2}$
Addendum	The Circular Pitch	Multiply the Circular Pitch by 0·3183 or $a=\frac{d}{t}$	$a=0{\cdot}3183p$
Dedendum	The Circular Pitch	Multiply the Circular Pitch by 0·3683	$b=0{\cdot}3683p$
Working Depth	The Circular Pitch	Multiply the Circular Pitch by 0·6366	$2a=0.6366p$
Whole Depth	The Circular Pitch	Multiply the Circular Pitch by 0·6866	$a+b=0{\cdot}6866p$
Clearance	The Circular Pitch	Multiply the Circular Pitch by 0·05	$c=0{\cdot}05p$
Clearance	Thickness of Tooth	One-tenth the Thickness of Tooth at Pitch Line	$c=\frac{g}{10}$

These formulæ apply in cases where the addendum is equal to $0{\cdot}3183p$.

DIAMETRAL PITCH

Diametral Pitch is the Number of Teeth to each Inch of the Pitch Diameter.

To Find	*Having*	*Rule.*	*Formula.*
The Diametral Pitch	The Circular Pitch	Divide 3·1416 by the Circular Pitch	$P=\frac{3·1416}{p}$
The Diametral Pitch	The Pitch Diameter and Number of Teeth	Divide Number of Teeth by Pitch Diameter	$P=\frac{t}{d}$
The Diametral Pitch	The Outside Diameter and Number of Teeth	Divide Number of Teeth plus 2 by Outside Diameter	$P=\frac{t+2}{d}$
Pitch Diameter	The Number of Teeth and Diametral Pitch	Divide Number of Teeth by the D.P.	$d=\frac{t}{P}$
Pitch Diameter	The Number of Teeth and Outside Diameter	Divide the product of Outside Diameter and Number of Teeth by Number of Teeth plus 2	$t=\frac{jt}{t+2}$
Pitch Diameter	The Outside Diameter and Diametral Pitch	Subtract from the Outside Diameter the quotient of 2 divided by the D.P.	$d=j-\frac{2}{P}$
Pitch Diameter	Addendum and Number of Teeth	Multiply Addendum by the Number of Teeth	$d=at$
Outside Diameter	The Number of Teeth and Diametral Pitch	Divide Number of Teeth plus 2 by the D.P.	$j=\frac{t+2}{P}$
Outside Diameter	The Pitch Diameter and Diametral Pitch	Add to the Pitch Diameter the quotient of 2 divided by the D.P.	$j=d+\frac{2}{P}$

Outside Diameter	The Pitch Diameter and Number of Teeth	Divide the Number of Teeth plus 2 by the Number of Teeth and multiply by the Pitch Diameter.	$j=\frac{t+2}{t}d$
Outside Diameter	The Number of Teeth and Addendum	Multiply the Number of Teeth plus 2 by Addendum	$j=(t+2)a$
Number of Teeth	The Pitch Diameter and Diametral Pitch	Multiply Pitch Diameter by the D.P.	$t=dP$
Number of Teeth	The Outside Diameter and Diametral Pitch	Multiply Outside Diameter by the D.P. and subtract 2	$t=jP-2$
Thickness of Tooth	The Diametral Pitch	Divide 1·5708 by the D.P.	$g=\frac{1{\cdot}5708}{P}$
Addendum	The Diametral Pitch	Divide 1 by the D.P.	$a=\frac{1}{P}$ or $s=\frac{d}{t}$
Dedendum	The Diametral Pitch	Divide 1·157 by the D.P.	$b=\frac{1{\cdot}157}{P}$
Working Depth	The Diametral Pitch	Divide 2 by the D.P.	$2a=\frac{2}{P}$
Whole Depth	The Diametral Pitch	Divide 2·157 by the D.P.	$a+b=\frac{2{\cdot}157}{P}$
Clearance	The Diametral Pitch	Divide 0·157 by the D.P.	$c=\frac{0{\cdot}157}{P}$
Clearance	Thickness of Tooth	Divide Thickness of Tooth at Pitch Line by 10	$c=\frac{g}{10}$

These formulæ apply in cases where the addendum is equal to $\frac{1}{P}$.

Pulley Calculations: General Rule.—A general rule which is not affected by the system of gears and/or pulleys employed—it applies whether the train is simple or compound—is that the product of the diameters (or the number of teeth in the case of gears) of the driving wheel and the number of revolutions per minute of the first driver is equal to the product of the diameters or the number of teeth of the driven wheels and the number of revolutions per minute of the last driven wheel.

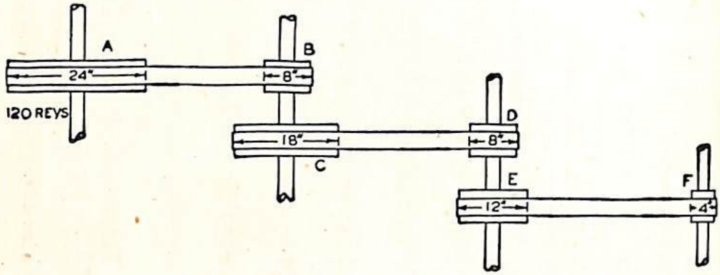


FIG. 104.—Example of triple belt drive, in which the main shaft revolves at 120 r.p.m. In this example $F = \frac{120 \times 24 \times 18 \times 12}{8 \times 8 \times 4} = 2{,}430$ r.p.m.

In the workshop, calculations regarding speeds of machine countershafts, etc., are frequent, and the following formulæ apply:

$$\text{R.P.M. of Driven Pulley or Gear} = \frac{\text{Diam. of Driver} \times \text{R.P.M. of Driver.}}{\text{Diam. of Driven.}}$$

$$\text{Diam. of Driven Pulley or Gear} = \frac{\text{Diam. of Driver} \times \text{R.P.M. of Driver.}}{\text{R.P.M. of Driven.}}$$

$$\text{R.P.M. of Driver Pulley or Gear} = \frac{\text{Diam. of Driven} \times \text{R.P.M. of Driven.}}{\text{Diam. of Driver.}}$$

$$\text{Diam. of Driver} = \frac{\text{Diam. of Driven} \times \text{R.P.M. of Driven.}}{\text{R.P.M. of Driver.}}$$

INDEX

A

Addendum, 21.
Angle, base spiral, 96.
— involute, function of, 98.
— pressure, 20, 27.
— spiral, 27.
Angular division tester, 88.
— velocity, 15.
Automobile gearbox, 9.
— gears, 68.
Axial feed, 62.
— pitch of worm, 38.

B

Ball and indicator, checking worm teeth by, 81.
— micrometer, checking worm teeth by, 82.
Base circle, 20, 95.
— pitch, 20.
— spiral angle, 96.
Bending stress, 113.
Bevel-gear units, 76.
— gears, 29, 121.
— — cutters for, 123.
— — double-helical, 33.
— — formulæ for, 121.
— — generation of, 31.
— — principles of, 29.
— — set over for, 124.
— — single-helical, generating, 56.
— — spiral, 13, 15, 32.
— — — generating, 57.
— — straight, 12.
— — — tooth form of, 32.
Blank diameter, determining size of, 22.
Block - centre chains, formula for, 132.
British Standard rack-form, 21.
— Standard system, 21.
Brown and Sharpe involute gear cutters, 121.
Built-up gears, 67.

C

Caliper, gear-tooth vernier, 78.
— Zeiss, 95.
Camshaft, gearing of, 11.
Chain sprocket, formula for, 132.
— wheels, block-centre, formula for, 135.
Chordal thickness, formula for, 98, 131.
Circular pitch, 21.
— — tables, 24, 136.
Cluster gears, cutting, 43.
Concentricity of teeth and bore, 83.
Contractions, strains, 66.
Cutters for mitre gears, 123.
— generating, making, 108

D

Dedendum, 21.
Diameter of blank, 20.
— of pitch circle, 20.
Diametral pitch, 22, 138.
Double-helical bevels, 33.
— — gears, 12.
— — meshing of, 25.
— — plano-generating, 53.

E

Efficiency of gears, 117.
Electric motors, speed of, 11.
End-mills, making, 103.
— -thrust, 27.
Epicyclic gear trains, 71.

F

Fellows gear generator, 42.
— process, 44.
Fly hob, 61.
Formate gears, 59.
Formed wheel, grinding, 50.
Formulæ for spur gearing, 120.
Friction, 117.

G

Gear cutters, B. and S., involute, 123.
— generation, 17, 41.
— generator, Fellows, 42.
— grinder, Maag, 47.
— single-reduction, 75.
— teeth, checking by comparison, 80.
— — checking shape of, 81.
— — helical, overlap of, 26.
— — measuring, 78.
— — comparative sizes of, 116.
— -tooth caliper, Sykes, 94.
— — — Zeiss, 94.
— — comparator, using, 90.
— vernier, 78.
— trains, epicyclic, 71.
— — velocity ratio of, 140.
— types, 12.
— units, bevel, 76.
— — worm, 77.
— wheel, definition of, 13.
— — form, 66.
— — types of, 66.
— units, spur, 75.
— — worm, 76, 77.
— working life of, 118.
— Zerol, 33.
Gearbox, 9.
Gears, applications of, 10.
— automobile, 68.
— bevel, 29.
— — cutters for, 123.
— — generating, 55
— — generation of, 31.
— built-up, 67.
— cause of noisy, 26.
— checking by running together, 89.
— chordal thickness of, 135.
— double-helical, meshing of, 25.
Gears, efficiency of, 117.
— formate, 59.
— helical, generating, 51.
— — hobbing, 52.
— — profile grinding, 55.
— hypoid, 59.
— load capacity of, 113.
— loading of, 14.
— lubrication of, 76, 118.
— measuring, 78.
— mitre, cutters for, 123.
— mounting, 74.
— requirements of, 14.
— root circle of, 20.
— screw, formula for, 126.
— shock loading of, 114.
— spiral, 13.
— — formula for, 126.
— — Gleason, 57.
— straight bevel, 12.
— — — tooth form of, 32.
— triple-helical, 12.
— turbine reduction, 68.
— types of, 9.
— velocity ratio of, 38.
— welded, 67.
Generating cutters, making, 108.
— helical gears, 51.
— rack, 23.
— single-helical bevels, 56.
— system, Sykes, 54.
Generation of bevel gears, 31.
— of gear, 17, 41.
— pitch circle of, 20.
Gleason spiral gear teeth, 57.
Grinding, formed wheel, 50.
— spur gears, 48.
— worms, 64.

H

Helical bevels, single, 33.
— gear, single, 11.
— — teeth, overlap of, 26.
— — units, 75.
— gears, direction of rotation of, 28.
— — double, 12.
— — generating, 51.
— — hobbing, 52.
— — plano-generating, 53.

Helical gears, profile grinding, 55.
— — spiral angle of, measuring, 88.
— — Sykes generating system, 54.
— — triple, 28.
Helicoidal worm thread, 36.
Hob, fly, 61.
Hobbing helical gears, 52.
— spur gears, 45.
Hobs, making, 100.
Hypoid gears, 60.

I

Internal spur gear, 44.
Involute curve, 19.
— function of angle, 96.
— gear cutters, B. and S., 121.
— tester, 83, 99.

L

Lead of worm, 38.
Load capacity of gears, 113.
Loading of gears, 14.
Lubrication of gears, 76, 118.

M

Maag spur gear grinder, 47.
Meshing of double-helical gears, 25.
Micrometer measurement of tooth thickness, 79.
Milling worm threads, 63.
Mitre gears, cutters for, 123.
Module pitch, formula for, 21.
Mounting gears, 74.

P

Parkson gear tester, 83.
Pinion, definition of, 13.
Pinions, making, 68.
Pitch, 21.
— base, 20.
Pitch circle, diameter of, 20.
— — of generation, 20.
— circular, 21, 136.
— cylinder of worm, 37.
— cylinders, 16.
— diametral, 22.
— line defined, 16.
— methods of defining, 21.
— module, formula for, 21.
— of worm, axial, 38.
— point defined, 16.
— testing, 86.
— — by optical method, 88.
— transverse, 21.
Pitches, tables of standard, 24.
Plano-generating machine, 41.
— processes, 53.
Pressure angle, 20, 27.
Profile grinding gears, 55.
— of teeth, checking, 83.
— of worm gears, checking, 84.
Proportions of teeth, 21.
Pulley calculations, 140.

R

Rack-form, British Standard basic, 21.
— generating, 23.
— method of generation, 18.
Roller Chain Wheels, 132.
Rollers, measuring tooth thickness by, 80.
Root circle of gear, 20.
— of tooth, fillet at, 31.
Rotation, direction of, 28.

S

Screw gears, formula for, 126.
Shafts and pinions of car gearboxes, 9.
Shock loading, 114.
Single-helical bevels, 33.
— gear, 11.
— -reduction gear units, 75.
Spiral angle, 27.
— — measuring, 88.
— bevel gears, 13, 15, 33.
— — — generating, 57.
— — Zerol, 33.
— gear, Gleason, 57.

Spiral gears, 14.
— — formula for, 126.
Sprocket wheels, formula for, 132.
Spur gear generator, Fellows, 42.
— — grinder, Maag, 47.
— — internal, 44.
— — units, 75.
— gears, definition of, 12.
— — generating, 41.
— — hobbing, 45.
Standard pitches, table of, 24.
Stress, bending, 113.
Sunderland process, 41.
Sykes generating system, 54.

T

Teeth, measuring by rollers, 80, 97.
Thread-milling, 63.
Thrust, end-, 27.
Tooth and bore, checking concentricity of, 83.
— depth of, 21.
— fillet at root of, 31.
— form of straight bevel gears, 32.
—proportions, 21.
— shape, checking, 81.
— thickness, finding from micrometer measurements, 97.
— — measurement of, 79.
Toothed gearing, 12.
Transverse pitch, 21.
Triple-helical gears, 12, 28.
Turbine reduction gears, 68.

U

Undercut, 21.

V

Velocity, angular, 15.
— ratio of gears, 38, 131.
Vernier, caliper, 78.

W

Welding gears, 67.
Worm and wheel, contact between, 39.
— dimensions of, 36.
— gear, 14.
— — units, 76, 77.
— gears, 16, 34.
— — checking profile of, 84.
— — dimensions of, 38.
— — tooth action of, 34.
— lead of, 38.
— pitch cylinder of, 37.
— teeth, checking by ball and indicator, 81.
— — — micrometer, 82.
— thread, helicoidal, 36.
— threads, milling, 63.
— wheel generator, 61.
— — pitch cylinder of, 16.
— wheels, cutting, 60.
— — formula for, 129.
— — producing, 60.
Worms, finishing, 65.
— formula for, 129.
— grinding, 64.

Z

Zeiss optical gear-tooth caliper, 94.
Zerol, gear, 33.
— spiral bevel, 33.

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